Theoretical Analysis of Lubrication Characteristics of Conical Bearings with Micropolar Fluid

Rajani C B[1], Hanumagowda B N[2], Vijayalaxmi S Shigehalli[3]

[1]*Department of Mathematics, Sri Siddhartha Institute of Technology, Tumkur, Karnataka, India.*
[2]*School of Applied Sciences, REVA Univesity, Bangalore, Karnataka, India*
[3]*Department of Mathematics, Rani Channamma University, Belagavi, Karnataka, India*

Abstract— **In this paper, on the basis of Eringen's micropolar fluid theory squeeze film lubrication between conical bearings lubricated using micropolar fluid is presented. The modified Reynold's equations governing the micropolar fluid is mathematically derived and analytical expressions for the film pressure, load and squeezing time at different values of microstructure measure parameter are obtained and the impact of microstructure is elaborated through various graphs. As per the outcomes obtained, the impact of non-Newtonian micropolar fluid increases the pressure, load carrying capacity and the squeezing time when compared with Newtonian lubricant case.**

Keywords— Squeeze film, Conical Bearing, Micropolar fluid, Reynolds equation

1. Introduction

In recent years, the squeeze film lubrication behaviour plays a useful part in industrial process dealing with bearings, machine parts, rolling elements, hydraulic systems and motorized engines. The squeezing performance changes the temperature, viscosity and density of the lubricants. Now a day, the increasing utilization of fluids containing micro-scale structures, for example, suspensions, added substances, long-chained polymers has gotten awesome consideration. Since, it is discovered that the Newtonian fluid constitutive calculation could not fulfill engineering request. Henceforth, the desire of engineers is a lubricant which acts as a defensive film and enables two surfaces by separating and smoothing. The usage of added substances in lubricants is a practice to get better performance of lubricants. The performance of non-Newtonian lubricant is, in which the shearing stress is no longer a constant. In this way it decreases the friction amongst them and consistently less warmth age in the machine, thereby keeping the working temperature of machine parts inside are at safe working points of confinement. Considering different bearings materials and operating conditions, the squeeze film lubrication were investigated by several authors such as slider bearings, rectangular plates, parallel stepped plates, curved circular plates, annular plates, conical bearings and rough conical plates [1]-[7]. To describe the additive effects several Microcontinuum theories have been suggested. The Micropolar fluids uncertainty can foresee conduct at microscale and revolution is freely clarified by a microrotation vector. Eringen's [8] is the principal examiner who put out the hypothesis to locate a numerical model of Micropolar fluids. Ariman et al., [9] presented a broad review of the Microcontinuum fluid theory. Zaheeruddin et al., [10] observed that the load carrying capacity reduces as the parameter characterizing the porosity enhances. Agrawal et al.,[11] and Verma et al.,[12] regarded that bearing supports much load when lubricated using a Micropolar fluid compared to a Newtonian fluid and the influence of porosity is to reduce the load capacity. Naduvinamani et al.,[13] using micropolar fluid examined the squeeze film characteristics of short partial porous journal bearings, Naduvinamani et al.,[14] presented porous inclined stepped bearing using micropolar fluid. Naduvinamani et al., [15] using micropolar fluid examined the squeeze film characteristics of finite porous journal bearings and examined that, the influence of Micropolar fluid considerably enhances the pressure and the load conveying capacity comparatively with Newtonian case. Lin et al,[16] investigated the squeeze film characteristics between a sphere and a plate lubricated by micropolar fluids. Abdallah et al., [17] analysed squeeze film characteristics between a sphere and a rough porous flat plate lubricated by micropolar fluids. Therefore various investigations are accounted for the Micropolar fluid lubrication and its applications.

2. Mathematical Analysis

The physical situation of the system of conical bearings lubricated by Micropolar fluid is illustrated in Figure1.The cone is having radius ‘*a*’ and angle 2θ and $V = -dh/dt$ represents the velocity of the fluid squeezing out of the cone housing with film thickness ‘*h*’.

Considering the standard assumptions of lubrication theory, the governing expressions of the Micropolar fluid film lubricant are in the form

Conservation of linear momentum:

$$\left(\mu+\frac{\chi}{2}\right)\frac{\partial^2 u}{\partial y^2}+\chi\frac{\partial v}{\partial y}=\frac{\partial p}{\partial r} \tag{1}$$

Conservation of annular momentum:

$$\gamma\frac{\partial^2 v_1}{\partial y^2}-\chi\frac{\partial u}{\partial y}-2\chi v_1=0 \tag{2}$$

Conservation of mass:

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{\partial v}{\partial y}=0 \tag{3}$$

Where μ is the coefficient of Newtonian viscosity, γ and χ are the coefficient of viscosity for Micropolar fluids. p is film pressure, (u, v) are the velocity components, v_1 is the microrotational velocity component

The appropriate boundary conditions of the bearings are expressed as:

i) At $y = hsin\theta$: $u = 0, v = sin\theta\frac{dh}{dt}, v_1 = 0$ (4a)

ii) At $y = 0$: $u = 0, v = 0, v_1 = 0$ (4b)

Integrating equation (1) and (3) and considering the boundary conditions (4a) and (4b) the solution for the velocity component is in the form

$$u=\frac{p^1}{\mu}\left[\frac{y^2}{2}-\frac{N^2 hsin\theta}{m}\frac{(Coshmy-1)}{Sinhmhsin\theta}\right]+ \\ \frac{D_1}{(1-N^2)}\left[y-\frac{N^2}{m}\left\{Sinhmy-(Coshmy-1)\frac{(Coshmhsin\theta-1)}{Sinhmhsin\theta}\right\}\right] \tag{5}$$

and

$$v_1=\frac{D_1}{(1-N^2)}(Coshmy-1)+ \\ \frac{Sinhmy}{Sinhmhsin\theta}\left[\frac{hsin\theta p^1}{2\mu}-\frac{D_1}{2(1-N^2)}(Coshmhsin\theta-1)\right]-\frac{yp^1}{2\mu} \tag{6}$$

Where

$$D_1=\frac{(1-N^2)}{2}\left[\frac{hsin\theta p^1}{\mu}\right] \text{ and } m=\frac{N}{l}, \; l=\left(\frac{\gamma}{4\mu}\right)^{1/2}$$

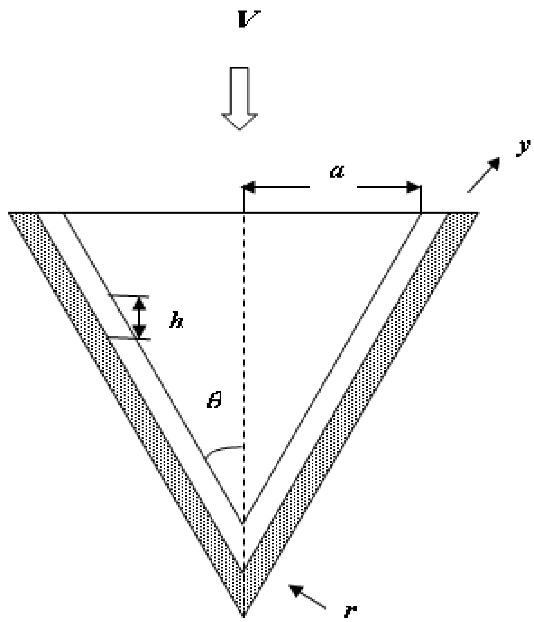

Figure 1: The Configuration of conical bearing of the problem

Integrating the equation (3) with y over film thickness considering 0 to $hsin\theta$, we get the modified form of Reynolds equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left[f\left(N,l,h,\theta\right)r\frac{dp}{dr}\right]=12\mu Sin\theta\frac{dh}{dt} \tag{7}$$

Where

$$f(N,l,h,\theta)=h^3Sin^3\theta+12l^2hSin\theta-6Nlh^2Sin^2\theta Coth\left(\frac{NhSin\theta}{2l}\right)$$

The pressure boundary conditions are:

$$\frac{dp}{dx}=0 \text{ at } r=0 \text{ and } p=0 \text{ at } r=Cosec\theta \tag{8}$$

Presenting the dimensionless quantities

$$h^*=\frac{h}{h_0}, l^*=\frac{l}{h_0}, r^*=\frac{r}{aCosec\theta},\ p^*=-\frac{ph_0^2}{\mu a^2\left(-\frac{dh}{dt}\right)Cosec\theta}$$

The non-dimensional form of Equation (7) is

$$\frac{\partial}{\partial r^*}\left[f(N,l^*,h^*,\theta)r^*\frac{dp^*}{dr^*}\right]=-12r^* \tag{9}$$

Where

$$f^*(N,l^*,h^*,\theta)=h^{*3}Sin^3\theta+12l^{*2}h^*Sin\theta-6Nl^*h^{*2}Sin^2\theta Coth\left(\frac{Nh^*Sin\theta}{2l^*}\right)$$

the boundary conditions (8) in form of non-dimensional are considered as:

i) at $r^*=0$: $\frac{dp^*}{dx^*}=0$ (10a)

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ii) at $r^* = 1$: $p^* = 0$ (10b)

Integrating equation (9) and using the corresponding condition (10a) and (10b) gives the approximate film pressure p^* in the form

$$p^* = -\frac{3(r^{*2}-1)}{f^*(N,l^*,h^*,\theta)} \tag{11}$$

The load supporting capacity is considered as

$$W = \int_0^{aCo\sec\theta} 2\pi rp\ dr \tag{12}$$

Using film pressure in the above equation and carrying out the integration, non-dimensional load W^* is found

$$W^* = \frac{Wh_0^3}{\mu a^3 (dh/dt) Co\sec^2\theta} = -\frac{4\pi}{f^*(N,l^*,h^*,\theta)} \tag{13}$$

The squeezing time is

$$t^* = \frac{th_0^2}{\mu a^3 (dh/dt) Co\sec^2\theta} = \int_{h_f^*}^{1} \frac{4\pi}{f^*(N,l^*,h^*,\theta)} dh^* \tag{14}$$

3. Results and Discussions

The results are examined for distinct dimensionless parameters. The effect of pressure, load and squeeze film time are graphically discussed.

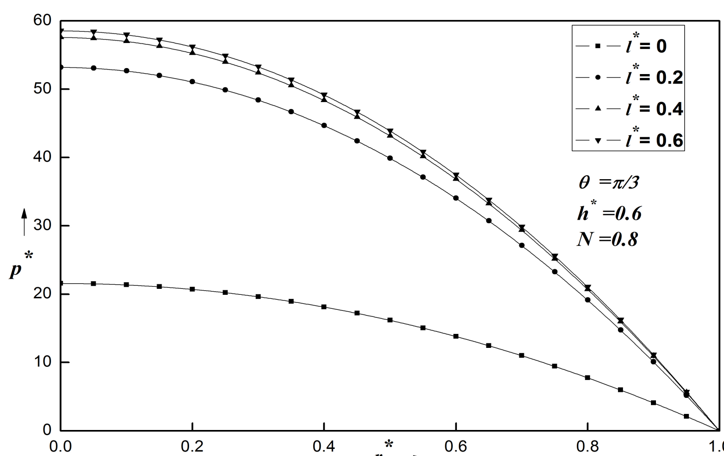

Figure 2 : Variation of p^* against r^* for different values l^* of with h^*=0.6, N=0.8 and $\theta = \pi/3$

3.1 Squeeze film pressure

The impact of micropolarity over variation of p^* against r^* for many values of l^* at $h^* = 0.6$, $N = 0.8$ and $\theta = \pi/3$ is illustrated in Figure 2. One can see that p^* increases with the increasing values of r^* and l^*. Figure 3 depicts p^* against r^* using many values of θ along $h^* = 0.6$, $N = 0.8$ and $l^* = 0.3$. The figure illustrates that, the dimensionless pressure

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reduces with increasing θ values. Figure 4 explains the variation of p^* against r^* using various N values at $\theta = \pi/3$, $l^* = 0.3$ and $h^* = 0.6$. We observe that with the increase of N values the pressure increases.

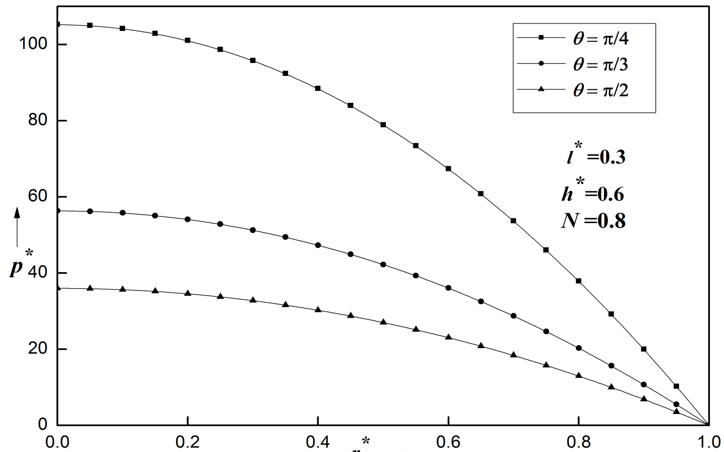

Figure 3 : Variation of p^* against r^* for different values θ of with $l^* = 0.3$, $h^* = 0.6$ and $N = 0.8$

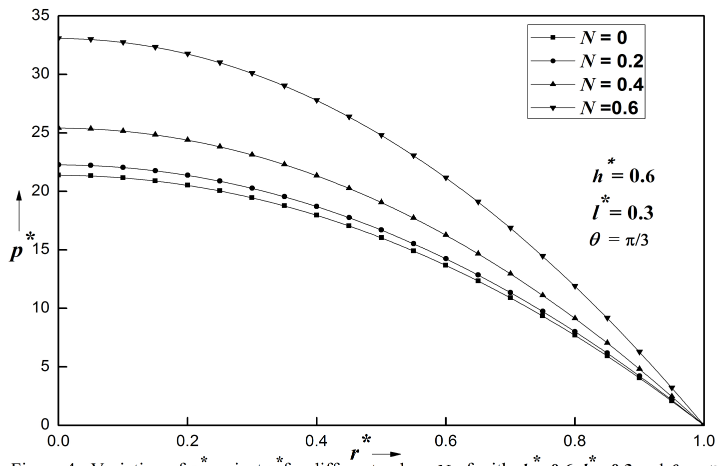

Figure 4 : Variation of p^* against r^* for different values N of with $h^* = 0.6$, $l^* = 0.3$ and $\theta = \pi/3$

3.2 Load Carrying Capacity

The variation of W^* against h^* for different l^* values with $\theta = \pi/3$ and $N = 0.8$ is seen in Figure 5. It shows that, W^* reduces with increasing h^* values and increases with increasing l^* values. Figure 6 illustrates the variation of W^* against h^* at distinct θ values of with $N = 0.8$ and $l^* = 0.3$ .It is seen that W^* reduces with the increase of θ values. Figure 7 portrays W^* against h^* at distinct values of N at $l^* = 0.3$ and $\theta = \pi/3$, it shows that, load W^* increases with enhancing values of N.

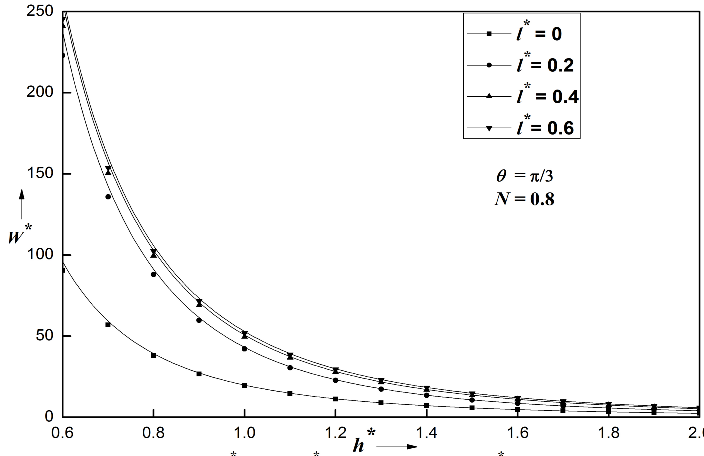

Figure 5 : Variation of W^* against h^* for different values l^* of with N=**0.8** and $\theta = \pi/3$

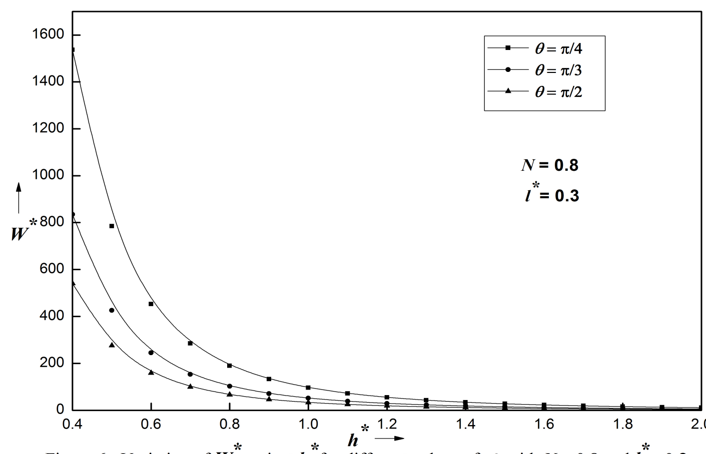

Figure 6 : Variation of W^* against h^* for different values of θ with N = **0.8** and l^* = **0.3**

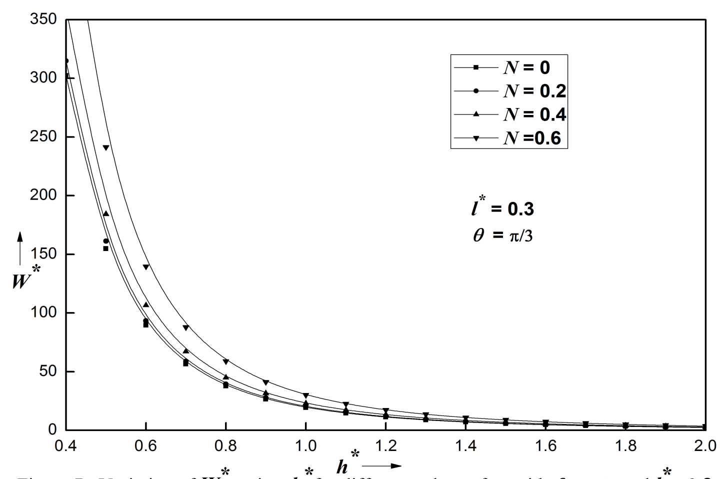

Figure 7 : Variation of W^* against h^* for different values of N with $\theta = \pi/3$ and l^* = **0.3**

3.3 Squeeze Film Time

Figure 8 exhibits the variation of t^* against h_f^* at distinct l^* values at N= 0.8 and θ = π/3 and one can observe that, t^* reduces with increasing h_f^* values and increases with increasing l^* values. In the Figure 9 the plot of t^* against h_f^* for distinct θ values at N = 0.8 and l^* = 0.3 is presented and is seen that t^* decreases with increasing values of θ. Figure 10 describes the deviation of t^* against h_f^* for distinct values of N at l^* = 0.3 and θ = π/3. Figure shows clearly that t^* rises with increase in N values.

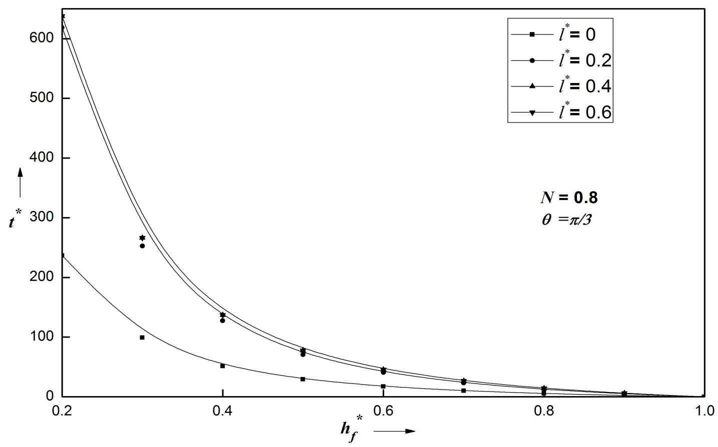

Figure 8 : Variation of t^* against h_f^* for different values l^* of with N=0.8 and θ =π/3

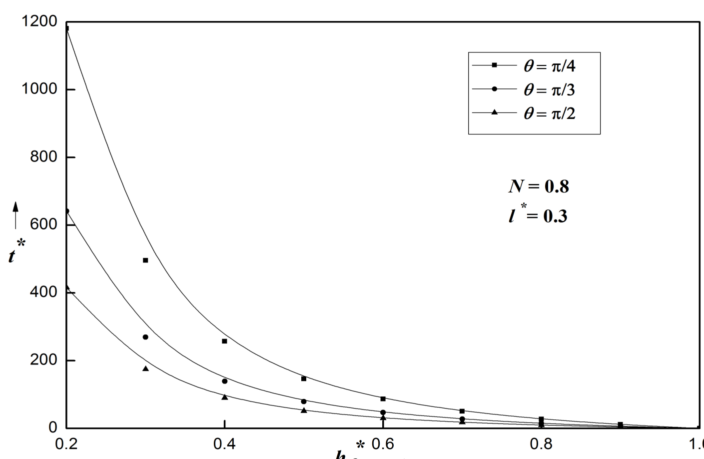

Figure 9 : Variation of t^* against h_f^* for different values θ of N=0.8 and l^*=0.3

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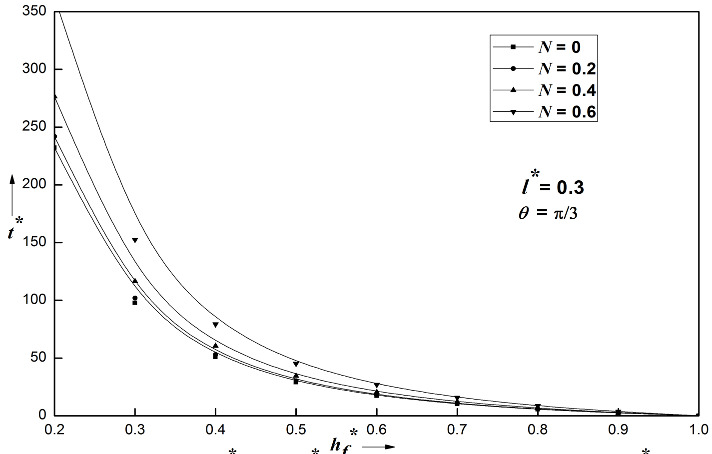

Figure 10 : Variation of t^* against h_f^* for different values of N with $\theta = \pi/3$ and $l^* = 0.3$

4. CONCLUSIONS

The impact of Micropolar fluid on squeeze film lubrication among conical bearings is examined. According to the study and the results discussed and presented as in the above section, the following conclusions are considered.

- It is found that for the conical bearings, the pressure, load and squeeze film time increases with the increase in l^* and N values.
- The pressure, load and squeezing time reduces with the increasing values of θ for the conical bearings.

Hence the outcomes so got are much useful for better performance and strength. As a result the present outcomes are accepted to be closer to the physical condition.

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