# Generalized Hyers-Ulam Stability of Non-Cubic Functional Equation in Random Normed Space Using Two Different Methods

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**Abstract:**

In this paper, we present the Hyers-Ulam stability of n-dimensional quadratic functional equation

$$f(3x+9y+27z)+f(3x-9y+27z)+f(3x+9y-27z)+f(-3x+9y+27z)=3[f(x)-f(-x)$$
$$+27[f(z)-f(-z)]+18[f(x)+f(-x)]+162[f(y)+f(-y)]+1458[f(z)+f(-z)]$$

and

$$g\left(\sum_{i=1}^{n} x_i\right)+\sum_{j=1}^{n} g\left(-x_j+\sum_{i=1, i\neq j}^{n} x_i\right)=(n-3)\sum_{1\le i<j\le n} g\left(x_i+x_j\right)+\left(-n^2+5n-2\right)\sum_{i=1}^{n} g\left(x_i\right)$$

in Random Normed Space.

**Keywords:** Fixed Point, Hyers-Ulam stability, Quadratic Functional Equation, Random Normed Space.

**MSC:** 39B52, 32B72, 32B82.

## 1. Introduction

In 1950, T. Aoki[1] generalized Hyers theorem for additive mappings. In 1978, Th. M. Rassias[14] proved a further generalization of Hyers Theorem by introducing the concept of the unbounded Cauchy difference for the sum of powers of two p-norms. Rassias established some linear mappings and concern problems in [11,13,14].

The functional equation

$$f(x+y)+f(x-y)=2f(x)+2f(y) \tag{1.1}$$

is called a quadratic functional equation. In particular, every solution of the quadratic functional equation is said to be a quadratic mapping.

During the last three decades the stability Theorem of Th. M. Rassias[14] provided a lot of influence for the development of stability theory of a large variety of functional equations. This new concept is known today with the term Hyers-Ulam-Rassias stability for functional equations. Following the innovative approach of Th. M. Rassias similar theorems where formulated and proved by a number of mathematicians. For example four years later in 1982, J. M. Rassias [9,10] proved a similar theorem for the case the unbounded Cauchy difference is the product of two p-norms. Recently, some mathematicians were investigate the quadratic functional equations in various normed space[5,8,12,15,16].

In this paper, the authors investigate the generalized Hyers-Ulam stability of a new type of n-dimensional quadratic functional equation

$$g\left(\sum_{i=1}^{n} x_i\right)+\sum_{j=1}^{n} g\left(-x_j+\sum_{i=1, i\neq j}^{n} x_i\right)=(n-3)\sum_{1\le i<j\le n} g\left(x_i+x_j\right)+\left(-n^2+5n-2\right)\sum_{i=1}^{n} g\left(x_i\right) \tag{1.2}$$

in Random Normed Space by using direct and fixed point method.

## 2. Preliminaries

In this section, we recall some notations and basic definitions used in this article.

**Definition 2.1** A mapping $T:[0,1]\times[0,1]\to[0,1]$ is called a continuous triangular norm, if T satisfies the following condition:

a) T is commutative and associative;
b) T is continuous
c) $T(a,1)=a$ for all $a\in[0,1]$
d) $T(a,b)\le T(c,d)$ when $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

Typical examples of continuous t-norms are $T_p(a,b)=ab,\ T_m(a,b)=\min(a,b)$ and $T_L(a,b)=\max(a+b-1,0)$ ( The Lukasiewicz t-norm ). Recall [9] that if T is a t-norm and $x_n$ is a given sequence of numbers in [0,1], then $T_{i=1}^{n}x_{n+i}$ is defined recurrently by $T_{i=1}^{'}x_i=x_i$ and $T_{i=1}^{n}x_i=T\left(T_{i=1}^{n-1}x_i,x_n\right)$ for $n\ge 2$, $T_{i=1}^{\infty}x_i$ is defined as $T_{i=1}^{\infty}x_{n+i}$ . It is known that, for the Lukasiewicz t-norm, the following implication holds:

$$\lim_{n\to\infty}(T_L)_{i=1}^{\infty}x_{n+i}=1\Leftrightarrow\sum_{n=1}^{\infty}(1-x_n)<\infty$$

**Definition 2.2** A random normed space ( briefly, RN-Space ) is a triple $(X,\mu,T)$, where X is a vector space. T is a continuous t-norm and $\mu$ is a mapping from X into $D^+$ satisfies the following conditions:

(RN1) $\mu_x(t)=\varepsilon_0(t)$ for all $t>0$ if and only if $x=0$.

(RN2) $\mu_{\alpha x}(t)=\mu_x\left(\frac{t}{|\alpha|}\right)$for all $x\in X$ , and $\alpha\in\Re$ with $\alpha\ne 0$.

(RN3) $\mu_{x+y}(t+s)\ge T\left(\mu_x(t),\mu_y(s)\right)$for all $x,y\in X\quad and\ t,s\ge 0$.

**Definition 2.3** Let $(X,\mu,T)$be a RN-space.

1) A sequence $\{x_n\}$ in X is said to be convergent to a point $x\in X$ if, for any $\varepsilon>0$ *and* $\lambda>0$, there exists a positive integer N such that $\mu_{x_n-x}(\varepsilon)>1-\lambda$ for all $n>N$.
2) A sequence $\{x_n\}$in X is called a Cauchy sequence if, for any $\varepsilon>0$ *and* $\lambda>0$, there exists a positive integer N such that $\mu_{x_n-x_m}(\varepsilon)>1-\lambda$ for all $n\ge m\ge N$.
3) A RN-Space $(X,\mu,T)$is said to be complete, if every Cauchy sequence in X is convergent to a point in X.

All over this paper we use the following notation for a given mapping $Q:X\to Y$ as

$$Q(x_1,x_2,...,x_n)=g\left(\sum_{i=1}^{n}x_i\right)+\sum_{j=1}^{n}g\left(-x_j+\sum_{i=1,i\ne j}^{n}x_i\right)-(n-3)\sum_{1\le i<j\le n}g\left(x_i+x_j\right)-\left(-n^2+5n-2\right)\sum_{i=1}^{n}g\left(x_i\right)$$

for all $x_1,x_2,...,x_n\in X$ .

**3. Random Stability Results: Direct Method**

In this section, the generalized Ulam-Hyers Stability of the functional equation (1.2) in RN-Space is provided. All through this section, let us consider X be a linear space $(Y,\mu,T)$ is a complete RN-Space.

**Theorem 3.1** Let $j=\pm 1$ , $Q:X\to Y$ be a mapping for which there exists a function $\eta:X^n\to D^+$ with the condition

$$\lim_{k\to\infty} T_{i=0}^{\infty}\left(\eta_{2^{(k+i)}x_1, 2^{(k+i)}x_2, \ldots, 2^{(k+i)}x_n}\left(2^{2(k+i+1)j}t\right)\right) = 1 \quad (3.1)$$

$$= \lim_{k\to\infty} \eta_{2^{kj}x_1, 2^{kj}x_2, \ldots, 2^{kj}x_n}\left(2^{2kj}t\right) \quad (3.2)$$

such that the functional inequality with $f(0)=0$ such that

$$\mu_{Q(x_1,x_2,\ldots,x_n)}(t) \geq \eta_{(x_1,x_2,\ldots,x_n)}(t) \quad (3.3)$$

for all $x_1, x_2, \ldots, x_n \in X$ and all $t>0$. Then there exists a unique quadratic mapping $A: X \to Y$ satisfies the functional equation (1.2) and

$$\mu_{A(x)-g(x)}(t) \geq T_{i=0}^{\infty}\left(\eta_{2^{(i+1)j}x, -2^{(i+1)j}x, 2^{(i+1)j}x, -2^{(i+1)j}x, 2^{(i+1)j}x, 0, \ldots, 0}\left(2^{2(i+1)j}t\right)\right) \quad (3.4)$$

for all $x \in X$ and all $t>0$. The mapping $A(x)$ is defined by

$$\mu_{A(x)}(t) = \lim_{k\to\infty} \mu_{\frac{g(2^{kj}x)}{2^{2kj}}}(t) \quad (3.5)$$

for all $x \in X$ and all $t>0$.

**Proof.** Assume $j=1$. Setting $(x_1, x_2, \ldots, x_n)$ *by* $(x, -x, x, -x, x, 0, \ldots, 0)$ in (3.1), we acquire

$$\mu_{2(n-5)g(2x)-8(n-5)g(x)}(t) \geq \eta_{x,-x,x,-x,x,0,\ldots,0}(t) \quad (3.6)$$

for all $x \in X$ and all $t>0$. It follows from (3.5) and (RN2), we arrive

$$\mu_{\frac{g(2x)}{2^2}-g(x)}(t) \geq \eta_{x,-x,x,-x,x,0,\ldots,0}\left(2(n-5)2^2 t\right) \quad (3.7)$$

for all $x \in X$ and all $t>0$. Replacing $x$ *by* $2^k x$ in (3.6), we catch

$$\mu_{\frac{g(2^{k+1}x)}{2^{2(k+1)}}-\frac{g(2^k x)}{2^{2k}}}(t) \geq \eta_{2^k x, -2^k x, 2^k x, -2^k x, 2^k x, 0, \ldots, 0}\left(2^{2(k+1)}2(n-5)t\right) \quad (3.8)$$

$$\geq \eta_{x,-x,x,-x,x,0,\ldots,0}\left(\frac{2^{2(k+1)}2(n-5)}{\alpha^k}t\right)$$

for all $x \in X$ and all $t>0$. It follows from $\frac{g(2^n x)}{2^{2n}} - g(x) = \sum_{k=0}^{n-1} \frac{g(2^{k+1}x)}{2^{2(k+1)}} - \frac{g(2^k x)}{2^{2k}}$ and (3.8) that

$$\mu_{\frac{g(2^n x)}{2^{2n}}-g(x)}\left(t\sum_{k=0}^{n-1}\frac{\alpha^k}{2^{2(k+1)}2(n-5)}\right) \geq T_{k=0}^{n-1}\left(\eta_{x,-x,x,-x,x,0,\ldots,0}(t)\right) = \eta_{x,-x,x,-x,x,0,\ldots,0}(t)$$

(3.9)

$$\mu_{\frac{g(2^n x)}{2^{2n}}-g(x)}(t) \geq \eta_{x,-x,x,-x,x,0,\ldots,0}\left(\frac{t}{\sum_{k=0}^{n-1}\frac{\alpha^k}{2(n-5)2^{2(k+1)}}}\right) \quad (3.10)$$

for all $x \in X$ and all $t > 0$. Replacing $x$ *by* $2^m x$ in (3.10), we arrive that

$$\mu_{\frac{g(2^{n+m} x)}{2^{2(n+m)}}-\frac{g(2^m x)}{2^{2m}}}(t) \geq \eta_{x,-x,x,-x,x,0,\ldots,0}\left(\frac{t}{\sum_{k=m}^{n+m}\frac{\alpha^k}{2(n-5)2^{2(k+1)}}}\right) \quad (3.11)$$

As $\eta_{x,-x,x,-x,x,0,\ldots,0}\left(\frac{t}{\sum_{k=m}^{n+m}\frac{\alpha^k}{2(n-5)2^{2(k+1)}}}\right) \to 1$ as $m,n \to \infty$, then $\left\{\frac{g(2^n x)}{2^{2n}}\right\}$ is a Cauchy sequence in $(Y,\mu,T)$. Since $(Y,\mu,T)$ is a complete RN-Space, this sequence converges to some point $A(x) \in Y$. Fix $x \in X$ and put $m = 0$ in (3.11), we have

$$\mu_{\frac{g(2^n x)}{2^{2n}}-g(x)}(t) \geq \eta_{x,-x,x,-x,x,0,\ldots,0}\left(\frac{t}{\sum_{k=0}^{n-1}\frac{\alpha^k}{2(n-5)2^{2(k+1)}}}\right) \quad (3.12)$$

and so, for every $\delta > 0$, we collect

$$\mu_{A(x)-g(x)}(t+\delta) \geq T\left(\mu_{A(x)-\frac{g(2^n x)}{2^{2n}}}(\delta), \mu_{\frac{g(2^n x)}{2^{2n}}-g(x)}(t)\right)$$

$$\geq T\left(\mu_{A(x)-\frac{g(2^n x)}{2^{2n}}}(\delta), \eta_{x,-x,x,-x,x,0,\ldots,0}\left(\frac{t}{\sum_{k=0}^{n-1}\frac{\alpha^k}{2(n-5)2^{2(k+1)}}}\right)\right) \quad (3.13)$$

Taking limit as $n \to \infty$ and using (3.13), we arrive

$$\mu_{A(x)-g(x)}(t+\delta) \geq \eta_{x,-x,x,-x,x,0,\ldots,0}\left(2(n-5)(2^2-\alpha)t\right) \quad (3.14)$$

Since $\delta$ was arbitrary, by taking $\delta \to 0$ in (3.14), we have

$$\mu_{A(x)-g(x)}(t) \geq \eta_{x,-x,x,-x,x,0,\ldots,0}\left(2(n-5)(2^2-\alpha)t\right) \quad (3.15)$$

Replacing $(x_1, x_2, \ldots, x_n)$ by $(2^n x_1, 2^n x_2, \ldots, 2^n x_n)$ in (3.3) respectively, we acquire

$$\mu_{Q(2^n x_1, 2^n x_2, \ldots, 2^n x_n)}(t) \geq \eta_{2^n x_1, 2^n x_2, \ldots, 2^n x_n}\left(2^{2n} t\right) \quad (3.16)$$

for all $x_1, x_2, ..., x_n \in X$ and for all $t > 0$. Since

$$\lim_{k\to\infty} T_{i=0}^{\infty}\left(\eta_{2^{(k+i)}x_1, 2^{(k+i)}x_2, ..., 2^{(k+i)}x_n}\left(2^{2(k+i+1)j}t\right)\right) = 1.$$

We conclude that A fulfils (1.1). To prove the uniqueness of the quadratic mapping A, assume that there exists a quadratic mapping B from X to Y, which satisfies (3.15). Fix $x \in X$. Clearly, $A(2^n x) = 2^{2n} A(x)$ and $B(2^n x) = 2^{2n} B(x)$ for all $x \in X$. It follows from (3.15) that

$$\mu_{A(x)-B(x)}(t) = \lim_{n\to\infty} \mu_{\frac{A(2^n x)}{2^{2n}} - \frac{B(2^n x)}{2^{2n}}}(t)$$

$$\mu_{\frac{A(2^n x)}{2^{2n}} - \frac{B(2^n x)}{2^{2n}}}(t) \geq \min\left\{\mu_{\frac{A(2^n x)}{2^{2n}} - \frac{g(2^n x)}{2^{2n}}}\left(\frac{t}{2}\right), \mu_{\frac{B(2^n x)}{2^{2n}} - \frac{g(2^n x)}{2^{2n}}}\left(\frac{t}{2}\right)\right\}$$

$$\geq \eta_{2^n x, -2^n x, 2^n x, -2^n x, 2^n x, 0, ..., 0}\left(2^{2n} 2(n-5)(2^2 - \alpha)t\right)$$

$$\geq \eta_{x,-x,x,-x,x,0,...,0}\left(\frac{2^{2n} 2(n-5)(2^2-\alpha)t}{\alpha^n}\right) \quad (3.17)$$

Since $\lim_{n\to\infty}\left(\frac{2^{2n} 2(n-5)(2^2-\alpha)t}{\alpha^n}\right) = \infty$, we get $\lim_{n\to\infty} \eta_{x,-x,x,-x,x,0,...,0}\left(\frac{2^{2n} 2(n-5)(2^2-\alpha)t}{\alpha^n}\right) = 1$. Therefore, it follows that $\mu_{A(x)-B(x)}(t) = 1$ for all $t > 0$ and so $A(x) = B(x)$. This completes the proof.

The following corollary is an immediate consequence of Theorem 3.1, concerning the stability of (1.2).

**Corollary 3.2**. Let $\varepsilon$ and s be non-negative real numbers. Let a quadratic function $Q : X \to Y$ satisfies the inequality

$$\mu_{Q(x_1, x_2, ..., x_n)}(t) \geq \begin{cases} \eta_{\varepsilon}(t) \\ \eta_{\varepsilon \sum_{i=1}^{n} \|x_i\|^s}(t); & s \neq 2 \\ \eta_{\varepsilon\left(\prod_{i=1}^{n} \|x_i\|^s + \sum_{i=1}^{n} \|x_i\|^{ns}\right)}(t); & s \neq \frac{2}{n} \end{cases} \quad (3.18)$$

for all $x_1, x_2, ..., x_n \in X$ and all $t > 0$. The there exists a unique quadratic function $A: X \to Y$ such that

$$\mu_{g(x)-A(x)}(t) \geq \begin{cases} \eta_{\frac{\varepsilon}{|6|(n-5)}}(t) \\ \eta_{\frac{\varepsilon\|x\|^s}{2(n-5)|2^2-2^s|}}(t) \\ \eta_{\frac{\varepsilon\|x\|^{ns}}{2(n-5)|2^2-2^{ns}|}}(t) \end{cases} \tag{3.19}$$

for all $x \in X$ and all $t > 0$.

**4. Random Stability Results: Fixed Point Method.**

In this section, the authors present the generalized Ulam-Hyers Stability of the functional equation (1.2), in Random Normed Space using fixed point method.

**Theorem 4.1** Let $Q: X \to Y$ be a mapping for which there exists a function $\eta: X^n \to D^+$ with the condition

$$\lim_{k\to\infty} \eta_{\delta_i^k x_1, \delta_i^k x_2, ..., \delta_i^k x_n}\left(\delta_i^{2k} t\right) = 1 \tag{4.1}$$

for all $x_1, x_2, ..., x_n \in X$ and all $t > 0$ and where $\delta_i = \begin{cases} 2, & i = 0; \\ \frac{1}{2} & i = 1; \end{cases}$ satisfying the functional inequality

$$\mu_{Q(x_1, x_2, ..., x_n)}(t) \geq \eta_{x_1, x_2, ..., x_n}(t) \tag{4.2}$$

for all $x_1, x_2, ..., x_n \in X$ and all $t > 0$. If there exists $L = L(i)$ such that the function

$$x \to \beta(x,t) = \eta_{\frac{x}{2}, -\frac{x}{2}, \frac{x}{2}, -\frac{x}{2}, \frac{x}{2}, 0, ..., 0}\left(2(n-5)t\right),$$

has the property, that

$$\beta(x,t) \leq L \frac{1}{\delta_i^2} \beta(\delta_i x, t) \tag{4.3}$$

for all $x \in X$ and $t > 0$. Then there exists a unique quadratic function $A: X \to Y$ satisfying the functional equation (1.2) and

$$\mu_{A(x)-g(x)}\left(\frac{L^{1-i}}{1-L} t\right) \geq \beta(x,t) \tag{4.4}$$

for all $x \in X$ and $t > 0$.

**Proof.** Let d be a general metric on $\Omega$, such that $d(a,b) = \inf\left\{k \in (0,\infty) / \mu_{(a(x)-b(x))}(kt) \geq \beta(x,t), x \in X, t > 0\right\}$. It is easy to see that

$(\Omega, d)$ is complete. Define $T:\Omega \to \Omega$ by $Ta(x) = \frac{1}{\delta_i^2} a(\delta_i x)$, for all $x \in X$. Now for $a, b \in \Omega$, we have $d(a,b) \le K$.

$$\Rightarrow \mu_{(a(x)-b(x))}(Kt) \ge \beta(x,t)$$
$$\Rightarrow \mu_{(Ta(x)-Tb(x))}\left(\frac{Kt}{\delta_i^2}\right) \ge \beta(x,t)$$
$$\Rightarrow d(Ta(x), Tb(x)) \le KL$$
$$\Rightarrow d(Ta, Tb) \le Ld(a,b) \qquad (4.5)$$

for all $a, b \in \Omega$. Therefore T is strictly contractive mapping on $\Omega$ with Lipschitz constant L. It follows from (3.6) that

$$\mu_{2(n-5)g(2x)-8(n-5)g(x)}(t) \ge \eta_{x,-x,x,-x,x,0,\ldots,0}(t) \qquad (4.6)$$

for all $x \in X$. It follows from (4.6) that

$$\mu_{\frac{g(2x)}{2^2}-g(x)}(t) \ge \eta_{x,-x,x,-x,x,0,\ldots,0}(8(n-5)t) \qquad (4.7)$$

for all $x \in X$. Using (4.3) for the case $i = 0$, it reduce to

$$\mu_{\frac{g(2x)}{2^2}-g(x)}(t) \ge L\beta(x,t)$$

for all $x \in X$. Hence, we obtain

$$d\left(\mu_{Tg(x)-g(x)}\right) \ge L = L^{1-i} < \infty \qquad (4.8)$$

for all $x \in X$. Replacing $x$ by $\frac{x}{2}$ in (4.7), we get

$$\mu_{\frac{g(x)}{2^2}-g\left(\frac{x}{2}\right)}(t) \ge \eta_{\frac{x}{2},-\frac{x}{2},\frac{x}{2},-\frac{x}{2},\frac{x}{2},0,\ldots,0}(8(n-5)t) \qquad (4.9)$$

for all $x \in X$. Using (4.3) for the case $i = 1$, it reduce to

$$\mu_{2^2 g\left(\frac{x}{2}\right)-g(x)}(t) \ge \beta(x,t) \Rightarrow \mu_{Tg(x)-g(x)}(t) \ge \beta(x,t)$$

for all $x \in X$. Hence, we get

$$d\left(\mu_{Tg(x)-g(x)}\right) \ge L = L^{1-i} < \infty \qquad (4.10)$$

for all $x \in X$. From (4.8) and (4.10), we can conclude

$$d\left(\mu_{Tg(x)-g(x)}\right) \ge L = L^{1-i} < \infty \qquad (4.11)$$

for all $x \in X$. In order to prove $A: X \to Y$ satisfies the functional equation (1.2), the remaining proof is similar by using Theorem 3.1. Since A is unique fixed point of T in the set $\Delta = \{g \in \Omega / d(g, A) < \infty\}$.Finally, A is an unique function such that

$$\mu_{g(x)-A(x)}\left(\frac{L^{1-i}}{1-L}t\right) \ge \beta(x,t)$$

for all $x \in X$ and $t > 0$. This completes the proof of the Theorem.

From the Theorem 4.1, we obtain the following Corollary concerning the stability for the functional equation (1.2).

**Corollary 4.2.** Suppose that a function $Q: X \to Y$ satisfies the inequality

$$\mu_{Q(x_1,x_2,\ldots,x_n)}(t) \geq \begin{cases} \eta_{\varepsilon}(t) \\ \eta_{\varepsilon\sum_{i=1}^{n}\|x_i\|^s}(t); & s \neq 2 \\ \eta_{\varepsilon\left(\prod_{i=1}^{n}\|x_i\|^s + \sum_{i=1}^{n}\|x_i\|^{ns}\right)}(t); & s \neq \frac{2}{n} \end{cases} \tag{4.12}$$

for all $x_1, x_2, \ldots, x_n \in X$ and all $t > 0$, where $\varepsilon, s$ are constants with $\varepsilon > 0$, then there exists a unique quadratic mapping $A: X \to Y$ such that

$$\mu_{g(x)-A(x)}(t) \geq \begin{cases} \eta_{\frac{\varepsilon}{(n-5)|6|}}(t) \\ \eta_{\frac{\varepsilon\|x\|^s}{2(n-5)|2^2-2^s|}}(t) \\ \eta_{\frac{\varepsilon\|x\|^{ns}}{2(n-5)|2^2-2^{ns}|}}(t) \end{cases} \tag{4.13}$$

for all $x \in X$ and all $t > 0$.

**Proof.** Setting

$$\mu_{Q(x_1,x_2,\ldots,x_n)}(t) \geq \begin{cases} \eta_{\varepsilon}(t) \\ \eta_{\varepsilon\sum_{i=1}^{n}\|x_i\|^s}(t), \\ \eta_{\varepsilon\left(\prod_{i=1}^{n}\|x_i\|^s + \sum_{i=1}^{n}\|x_i\|^{ns}\right)}(t), \end{cases}$$

for all $x_1, x_2, \ldots, x_n \in X$ and all $t > 0$. Then

$$\eta_{\left(\delta_i^k x_1, \delta_i^k x_2, \ldots, \delta_i^k x_n\right)}\left(\delta_i^{2k} t\right) = \begin{cases} \eta_{\varepsilon \delta_i^{2k}}(t) \\ \eta_{\varepsilon \sum_{i=1}^{n} \|x_i\|^s \delta_i^{(2-s)k}}(t) \\ \eta_{\varepsilon\left(\prod_{i=1}^{n} \|x_i\|^s \delta_i^{(2-s)k} + \sum_{i=1}^{n} \|x_i\|^{ns} \delta_i^{(2-ns)k}\right)}(t) \end{cases}$$

$$= \begin{cases} \to 1 \ \ as \ \ k \to \infty \\ \to 1 \ \ as \ \ k \to \infty \\ \to 1 \ \ as \ \ k \to \infty \end{cases}$$

But we have $\beta(x,t) = \eta_{\frac{x}{2}, -\frac{x}{2}, \frac{x}{2}, -\frac{x}{2}, \frac{x}{2}, 0, \ldots, 0}\left(2(n-5)t\right)$ has the property $L\frac{1}{\delta_i^2}\beta(\delta_i x, t)$ for all $x \in X$ and $t > 0$. Now

$$\beta(x,t) = \begin{cases} \eta_{\frac{\varepsilon}{2(n-5)}}(t) \\ \eta_{\frac{\varepsilon \|x\|^s}{2(n-5)2^s}}(t) \\ \eta_{\frac{\varepsilon \|x\|^{ns}}{2(n-5)2^{ns}}}(t) \end{cases}$$

$$L\frac{1}{\delta_i^2}\beta\left(\delta_i x, t\right) = \begin{cases} \eta_{\delta_i^{-2}\beta(x)}(t) \\ \eta_{\delta_i^{s-2}\beta(x)}(t) \\ \eta_{\delta_i^{ns-2}\beta(x)}(t) \end{cases}$$

By using Theorem 4.1, we prove the following six cases:

$L = 2^{-2}$ if $i = 0$ and $L = 2^{2}$ if $i = 1$

$L = 2^{s-2}$ for $s < 2$ if $i = 0$ and $L = 2^{2-s}$ for $s > 2$ if $i = 1$

$L = 2^{ns-2}$ for $s < \frac{2}{n}$ if $i = 0$ and $L = 2^{2-ns}$ for $s > \frac{2}{n}$ if $i = 1$

**Case .1:** $L = 2^{-2}$ if $i = 0$

$$\mu_{g(x)-A(x)}(t) \geq L\frac{1}{\delta_i^2}\beta\left(\delta_i x, t\right)(t) \geq \eta_{\left(\frac{\varepsilon}{6(n-5)}\right)}(t)$$

**Case.2:** $L = 2^2$ if $i = 1$

$$\mu_{g(x)-A(x)}(t) \geq L\frac{1}{\delta_i^2}\beta\left(\delta_i x, t\right)(t) \geq \eta_{\left(\frac{\varepsilon}{-6(n-5)}\right)}(t)$$

**Case.3:** $L = 2^{s-2}$ for $s < 2$ if $i = 0$

$$\mu_{g(x)-A(x)}(t) \geq L\frac{1}{\delta_i^2}\beta\left(\delta_i x, t\right)(t) \geq \eta_{\left(\frac{\varepsilon\|x\|^s}{2(n-5)\left(2^2-2^s\right)}\right)}(t)$$

**Case.4:** $L = 2^{2-s}$ for $s > 2$ if $i = 1$

$$\mu_{g(x)-A(x)}(t) \geq L\frac{1}{\delta_i^2}\beta\left(\delta_i x, t\right)(t) \geq \eta_{\left(\frac{\varepsilon\|x\|^s}{2(n-5)\left(2^s-2^2\right)}\right)}(t)$$

**Case.5:** $L = 2^{ns-2}$ for $s < \frac{2}{n}$ if $i = 0$

$$\mu_{g(x)-A(x)}(t) \geq L\frac{1}{\delta_i^2}\beta\left(\delta_i x, t\right)(t) \geq \eta_{\left(\frac{\varepsilon\|x\|^{ns}}{2(n-5)\left(2^2-2^{ns}\right)}\right)}(t)$$

**Case.6:** $L = 2^{2-ns}$ for $s > \frac{2}{n}$ if $i = 1$

$$\mu_{g(x)-A(x)}(t) \geq L\frac{1}{\delta_i^2}\beta\left(\delta_i x, t\right)(t) \geq \eta_{\left(\frac{\varepsilon\|x\|^{ns}}{2(n-5)\left(2^{ns}-2^2\right)}\right)}(t)$$

Hence the proof is complete.

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