A new ranking approach for ordering of single valued triangular neutrosophic numbers

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Abstract:

Neutrosophic set and neutrosophic number are renowned theories to deal with complex, not clearly explained and uncertain real life problems. In this paper we proposed a new ranking method by using middle point of two neutrosophic numbers. Properties for the ordering are also studied. The method is illustrated by numerical examples and compared with other methods.

Keywords: Neutrosophic numbers, single valued neutrosophic numbers, triangular neutrosophic numbers, ranking neutrosophic numbers

1. Introduction

Theory of uncertainty plays an important role in practical life problems and engineering problems. So researchers showed interest to represent uncertainty in mathematical modeling. To handle uncertainty Zadeh[1] first introduced fuzzy sets. Atanassov [2] extended the fuzzy sets to intuitionistic fuzzy sets to handle uncertainty and vagueness. The fuzzy sets are represented by membership function whereas intuitionistic fuzzy sets are represented by membership and non membership function. Fuzzy systems and intuitionistic fuzzy sets cannot successfully deal with uncertainty. Therefore some novel theories are mandatory for solving the problem with uncertainty. To deal with uncertainty more precisely, Smarandache [3]initiated the notion of neutrosophic set(NS),a genaralised version of classical set, fuzzy set, intuitionistic fuzzy set etc. In the neutrosophic logic, each parameter is estimated by a triplet viz., truth function, indeterminacy function and falsity function. The indeterminacy function which is used to represent uncertain data which is not possible in fuzzy and intuitionistic set plays an important role to get accurate function. Ranking of fuzzy numbers, intuitionistic fuzzy numbers and neutrosophic numbers are very important because of its application in decision making and optimization even in developing of various mathematical structures. Till now several ranking methods have been developed to rank fuzzy and intuitionistic fuzzy numbers. Deli and Subas [4] introduced a ranking method to rank single valued neutrosophic number. Deli [5] introduced a ranking method using score function to rank trapezoidal neutrosophic numbers based on Einstein operators. Different forms of triangular neutrosophic numbers, deneutrosophication technique was given by Avishek chakraborty[6] . He also applied the ranking method to critical path analysis. Said Broumi[7] ordered triangular neutrosophic numbers using score functions and accuracy functions and applied the same to shortest path problem. Sumathi[8] explored differential equation in neutrosophic environment and solved second order linear differential equation with trapezoidal neutrosophic number as boundary condition. Samah Ibrahim Abdel Aal[9] introduced two ranking method to rank triangular neutrosophic number.

Regarding to several ranking method having been reviewed above, they are based on score function and value ambiguity method. Since each of the technique have some drawbacks and complicated to use we introduce a new method to rectify the drawbacks and our method is simple and easy to use.

Structure of the paper:

The paper is organized as follows. In Section 1, the basic concept on neutrosophic set theory and the existing ranking method is discussed. Section 2 contains the preliminaries section. In Section 3 de-neutrosophication of triangular neutrosophic number is introduced and is ordered by using the middle pointof a neutrosophic number. In Section 4, numerical examples are given with comparison of the existing method. Conclusions are written in Section 5.

2. Preliminaries

Definition 2.1:[1]A fuzzy set C in X is defined as $C = \left\{\left(x, C_\mu(x)\right): x\epsilon X, C_\mu(x)\epsilon[0,1]\right\}$ and is generally represented as $\left(x, C_\mu(x)\right)$ and $C_\mu(x)$ is the membership function.

Definition 2.2:[2]An intuitionistic fuzzy set C in X is defined as $C = \{\left(x, C_\mu(x), C_{\gamma(x)}\right): x\epsilon X, C_\mu(x)\epsilon[0,1], C_\gamma(x)\epsilon[0,1]\}$ and is generally represented as $\left(x, C_\mu(x), C_\gamma(x)\right)$ and $C_\mu(x), C_\gamma(x)$ are the membership and non membership function respectively satisfying the relation $0 \leq C_\mu(x) + C_\gamma(x) \leq 1$.

Definition 2.3:[3] A neutrosophic set N in X is defined as $N = \{\left(x, N_T(x), N_I(x), N_F(x)\right): x\epsilon X\}$ where $N_T(x): X \to [0,1]$ is said to be the truth membership function ,$N_I(x): X \to [0,1]$ is said to be the indeterminacy membership function and $N_F(x): X \to [0,1]$ is said to be the falsity membership function and the relation $-0 \leq N_T(x) + N_I(x) + N_F(x) \leq 3+$ holds.

Definition 2.3:[10] A single valued A neutrosophic set N in X is defined as $N = \{\left(x, N_T(x), N_I(x), N_F(x)\right): x\epsilon X\}$where$N_T(x): X \to [0,1]$ is said to be the truth membership function ,$N_I(x): X \to [0,1]$ is said to be the indeterminacy membership function and $N_F(x): X \to [0,1]$ is said to be the falsity membership function and the relation $0 \leq supN_T(x) + supN_I(x) + supN_F(x) \leq 3$ holds.

3. Proposed ranking method for triangular neutrosophic number

In this section single valued triangular neutrosophic number is defined with its parametric form. Arithmetic operations on triangular neutrosophic number is discussed and we propose a new ranking method to rank triangular neutrosophic number by using its middle point.

Definition 3.1: A single valued triangular neutrosophic number is defined as $N = \{(x_{0,}\rho_1, \omega_1)(y_{0,}\rho_2, \omega_2)(z_{0,}\rho_3, \omega_3)$ whose truth, indeterminacy and falsity is as follows:

$$N_T(x) = \begin{cases} \frac{1}{\rho_1}(x - x_0 + \rho_1), \mathrm{x}_0 - \rho_1 \leq \mathrm{x} \leq \mathrm{x}_0 \\ 1, \quad x = x_0 \\ \frac{1}{\omega_1}(x_0 - x + \omega_1, x_0 \leq x \leq x_0 + \omega_1 \end{cases}$$

2

$$N_I(x) = \begin{cases} \frac{1}{\rho_2}(y_0 - x), & y_0 - \rho_2 \le x \le y_0 \\ 0, & x = y_0 \\ \frac{1}{\omega_2}(x - x_0), & y_0 \le x \le y_0 + \omega_2 \end{cases}$$

$$N_F(x) = \begin{cases} \frac{1}{\rho_3}(z_0 - x), & z_0 - \rho_2 \le x \le z_0 \\ 0, & x = z_0 \\ \frac{1}{\omega_3}(x - z_0), & z_0 \le x \le z_0 + \omega_3 \end{cases}$$

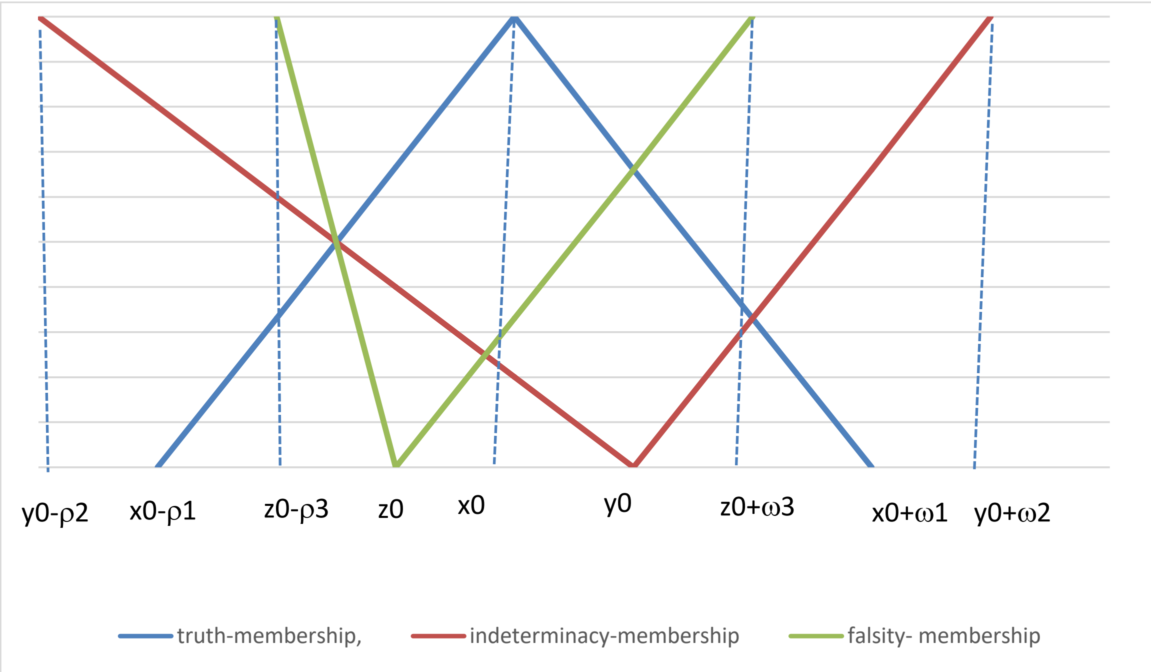

Fig 1 Representation of triangular neutrosophic number

Definition 3.2:A single valued triangular neutrosophic number $N = \{(x_0,\rho_1,\omega_1)(y_0,\rho_2,\omega_2)(z_0,\rho_3,\omega_3)$ in parametric form is given by $\{\left(\underline{N_T(r)}, \overline{N_T(r)}\right), \left(\underline{N_I(r)}, \overline{N_I(r)}\right), \left(\underline{N_F(r)}, \overline{N_F(r)}\right)\}$ with $0 \le r \le 1$ and

$\underline{N_T\ (r)} = x_0 - \rho_1 + \rho_1 r,$ $\quad \overline{N_T(r)} = x_0 + \omega_1 - \omega_1 r$

$\underline{N_I\ (r)} = y_0 - \rho_2 r,$ $\quad \overline{N_T(r)} = y_0 + \omega_2 r$

$\underline{N_F\ (r)} = z_0 - \rho_3 r,$ $\quad \overline{N_T(r)} = z_0 + \omega_3 r.$

3.3 Arithmetic operations of triangular neutrosophic number:

3.3.1 Addition of two triangular neutrosophic numbers:

For any two triangular neutrosophic numbers $N = \{(x_{0,}\rho_1,\omega_1)(y_{0,}\rho_2,\omega_2)(z_{0,}\rho_3,\omega_3)\}$ and

$S = \{(u_{0,}\tau_1, \varphi_1)(v_{0,}\tau_2, \varphi_2)(w_{0,}\tau_3, \omega_3)\}$ addition of N and M is given by

$$M + S = \{(x_0 + u_0, \rho_1 + \tau_1, \omega_1 + \varphi_1)(y_0 + v_0, \rho_2 + \tau_2, \omega_2 + \varphi_2)(z_0 + w_0, \rho_3 + \tau_3, \omega_3 + \varphi_3)\}$$

3.3.2 Image of triangular neutrosophic number:

For any triangular neutrosophic numbers $N = \{(x_{0,}\rho_1, \omega_1)(y_{0,}\rho_2, \omega_2)(z_{0,}\rho_3, \omega_3)\}$ image of N is given by $-N = \{(-x_{0,}\rho_1, \omega_1)(-y_{0,}\rho_2, \omega_2)(-z_{0,}\rho_3, \omega_3)\}$

3. 4 Middle point of neutrosophic number:

The middle point of triangular neutrosophic number is given by $N = \{(x_{0,}\rho_1, \omega_1)(y_{0,}\rho_2, \omega_2)(z_{0,}\rho_3, \omega_3)$ is given by

$$M(N) = \frac{1}{2}\int_0^1 (\underline{N_T(r)} + \overline{N_T(r)} + \underline{N_I(r)} + \overline{N_I(r)} + \underline{N_F(r)} + \overline{N_F(r)})dr$$

$$= \frac{1}{2}\int_0^1 (x_0 - \rho_1 + \rho_1 r + x_0 + \omega_1 - \omega_1 r + y_0 - \rho_2 r + y_0 + \omega_2 r + z_0 - \rho_3 r + z_0 + \omega_3 r)dr$$

$$= \frac{1}{4}[4x_0 - \rho_1 + \omega_1 + 4y_0 - \rho_2 + \omega_2 + 4z_0 - \rho_3 + \omega_3].$$

Theorem 3.4.1: Let$N = \{(x_{0,}\rho_1)(y_{0,}\rho_2)(z_{0,}\rho_3,)$ be a symmetric triangular neutrosophic number then $M(N) = [x_0 + y_0 + z_0]$.

Proof:

For a triangular neutrosophic number

$$M(N) = \frac{1}{2}\int_0^1 (\underline{N_T(r)} + \overline{N_T(r)} + \underline{N_I(r)} + \overline{N_I(r)} + \underline{N_F(r)} + \overline{N_F(r)})dr$$

$$= \frac{1}{2}\int_0^1 (x_0 - \rho_1 + \rho_1 r + x_0 + \rho_1 - \rho_1 r + y_0 - \rho_2 r + y_0 + \rho_2 r + z_0 - \rho_3 r + z_0 + \rho_3 r)dr$$

$$= [x_0 + y_0 + z_0]$$

Hence proved.

Theorem 3.4.2: For any two triangular neutrosophic numbers $N = \{(x_{0,}\rho_1, \omega_1)(y_{0,}\rho_2, \omega_2)(z_{0,}\rho_3, \omega_3)\}$ and $S = \{(u_{0,}\tau_1, \varphi_1)(v_{0,}\tau_2, \varphi_2)(w_{0,}\tau_3, \omega_3)\}$, then $M(N + S) = M(N) + M(S)$.

Proof: For a triangular neutrosophic number

$$M(N + S) = \frac{1}{2}\int_0^1 (\underline{N_T(r)} + \overline{N_T(r)} + \underline{N_I(r)} + \overline{N_I(r)} + \underline{N_F(r)} + \overline{N_F(r)} + \underline{S_T(r)} + \overline{S_T(r)} + \underline{S_I(r)} + \overline{S_I(r)} + \underline{S_F(r)} + \overline{S_F(r)})dr$$

$$= \frac{1}{2}\int_0^1 (\underline{N_T(r)} + \overline{N_T(r)} + \underline{N_I(r)} + \overline{N_I(r)} + \underline{N_F(r)} + \overline{N_F(r)})dr + \frac{1}{2}\int_0^1 (\underline{S_T(r)} + \overline{S_T(r)} + \underline{S_I(r)} + \overline{S_I(r)} + \underline{S_F(r)} + \overline{S_F(r)})dr$$

$$= M(N) + M(S).$$

3.5 Ranking procedure:

For any two triangular neutrosophic numbers N,S the ranking of N and S is defined as follows:

(i) $M(N) > M(S)$ if and only if $N > S$

(ii) $M(N) < M(S)$ if and only if $N < S$

(iii) $M(N) = M(S)$ if and only if $N = S$

4. Numerical examples

In this section some numerical examples were presented to illustrate the above proposed ranking method and comparison with existing method was also done.

Example 4.1: Consider the following triangular neutrosophic number.

$$A = \{(1,3,5)(.5,1.5,2.5)(1.2,2.7,4.5)\}$$

By proposed method, we have $M(A) = 3.13$.

Example4.2: Consider the following sets of triangular neutrosophic number

Set: 1 $A = \{(0.2,0.4,0.5)(0.3,0.5,0.6)(0.1,0.2,0.3)\}$

$B = \{(0.37,0.52,0.72)0(.02,0.06,0.15)(0.12,0.25,0.42)\}$

$C = \{(0.9,0.44,0.58)(0.06,0.12,0.25)(0.02,0.06,0.18)\}$

Set: 2 $A = \{(1,2,3)(0.5,1.5,2.5)(1.2,2.7,3.5)\}$

$B = \{(0.5,1.5,2.5)0(0.3,1.3,2.2)(0.7,1.7,2.2)\}$

$C = \{(0.3,1.2,2.8)(0.5,1.5,2.5)(0.8,1.7,2.7)\}$

Ranking of the set 1 and set 2 triangular neutrosophic numbers is discussed in table 1 and is also compared with the existing method. In the comparison we see that, the ranking procedure provided in [7], shows contradictive results than remaining methods. Our method gives exact ranking of single valued triangular neutrosophic numbers.

Method	**Triangular neutrosophic number**	**Set 1**	**Set 2**
Proposed method	A B C	.52 .43 .31 A>B>C	3.01 2.17 2.3 A>C>B
Method proposed in [7]	A B C	.57 .73 .84 C>B>A	-.675 -.441 -.533 B>C>A
Method proposed in[6]	A B C	35 .198 .26 A>C>B	2.00 1.45 1.533 A>C>B

Table 1.Comparison table for ranking triangular neutrosophic number with the existing method

6. Conclusion

In order to rank triangular neutrosophic number and to overcome drawbacks of the existing methods we proposed a new ranking method .Our ranking method has some mathematical properties and also does not imply much computational effort. We also used comparative examples to illustrate the advantages of the proposed method.

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