C-Lambda-Compact Spaces

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Abstract: *Viglino[13], introduced the family of C-compact spaces, showing that every continuous function from a C-compact spaces into a Hausdorff spaces is a closed function and that this class of spaces properly contains the class of compact spaces. In this present paper we study these spaces by considering* λ *-open sets introduced by Arenas et.al [1]. We also characterize their fundamental properties.*

Keywords: λ *-open,* λ *-continuous,* λ *-adherent point,* λ *-regular, filter base* λ *-compact, C-*λ *-compact.*

1. Introduction and Preliminaries

It is well known that the image of a compact space under a continuous function into a Hausdorff space is closed. If we denote by P the property that every continuous function from a topological space into a Hausdorff space is closed then the problem is whether underlying topological space having the property P is always compact. Viglino [14] resolved this problem in 1969 in the negative and substantiated his argument with an example. He simultaneously introduced a new class of topological space for which property P held. He called these spaces as C-compact. Since then, a tremendous number of papers such as Viglino[14],Sakai[13], Herringaton et.al.[8], Viglino[15], Goss & Viglino[7] and Kim[9] have appeared on C-compact spaces. Maki [3] introduced the notion of Λ-sets in topological spaces. A Λ-set is a set A which is equal to its kernel, that is, to the intersection of all open super sets of A. Arenas et.al. [1] introduced and investigated the notion of λ-closed sets by involving Λ-sets and closed sets. C.Duraisamy et.al [5] discussed some of the properties of λ-continuous functions.

Throughout this paper X and Y represents non-empty topological spaces on which no separation axioms are assumed, unless otherwise stated. For any subset A of X, Cl(A) and Int(A) respectively represents the closure and interior of A. Now we recall some definitions and results, which we have used in the sequel. In this paper, we venture to generalize C-compact spaces by using λ-open set and shall term them as C-λ-compact spaces.

Definition 1.1: Viglino[14] A topological space X is called C-compact if for each closed subset $A \subset X$ and for each open cover $\mathcal{U} = \{U_\lambda \setminus \lambda \in \Lambda\}$ of A, there exists a finite sub collection $\{U_{\lambda_i} \setminus 1 \le i \le n\}$ of U such that $A \subset \bigcup_{i=1}^{n} \mathrm{Cl}(U_{\lambda_i})$.

Definition 1.2: Njastad[12] A subset A of a topological space X is called α-open if A ⊆int(cl(int(A))). The complement of an α-open set is called an α-closed. Equivalently, a set F is α-closed in X if cl(int(cl(F))) ⊆ F. The family of all α-open (respectively α-closed) sets in X is denoted by αO(X) (respectively αC(X).

Remark 1.3: Mashour et.al.[11] Continuity implies α-continuity but not conversely.

Remark 1.4: Mashour et.al.[11] Every open mapping (closed mapping) is α-open (α-closed) but the converse is not true.

2. C-λ-COMPACT SPACES

Definition 2.1: A topological space X is said to be C-λ-compact if for each closed subset $A \subset X$ and for each λ-open cover $\mathcal{U} = \{U_\lambda \setminus \lambda \in \Lambda\}$ of A, there exists a finite sub collection $\{U_{\lambda_i} \setminus 1 \le i \le n\}$ of U such that, $A \subset \bigcup_{i=1}^{n} \mathrm{Cl}_\lambda(U_{\lambda_i})$.

Lemma 2.2: A topological space X is C-λ-compact if and only if for each closed subset $A \subset X$ and for each λ-regular open cover $\mathcal{U} = \{U_\lambda \setminus \lambda \in \Lambda\}$ of A, there exists a finite sub collection $\{U_{\lambda_i} \setminus 1 \le i \le n\}$ of U such that, $A \subset \bigcup_{i=1}^{n} \mathrm{Cl}_\lambda(U_{\lambda_i})$.

Proof: Let X be C-λ-compact and let $\{U_\lambda \setminus \lambda \in \Lambda\}$ be any cover of A by λ-open sets. Then $\mathcal{V} = \{\mathrm{Int}_\lambda(\mathrm{Cl}_\lambda(U_\lambda))\}$ is a λ-regular open cover of A and so there exists a finite sub collection

$\{\mathrm{Int}_\lambda(\mathrm{Cl}_\lambda(\mathrm{U}_{\lambda_i})): 1 \le i \le n\}$ of $\mathcal{V}$ such that $\mathrm{A} \subset \bigcup_{i=1}^{n} \mathrm{Cl}_\lambda\{\mathrm{Int}_\lambda(\mathrm{Cl}_\lambda(\mathrm{U}_{\lambda_i}))\}$. But for each *i*, we have $\mathrm{Cl}_\lambda\{\mathrm{Int}_\lambda(\mathrm{Cl}_\lambda(\mathrm{U}_{\lambda_i}))\} = \mathrm{Cl}_\lambda(\mathrm{U}_{\lambda_i})$

Therefore, $\mathrm{A} \subset \bigcup_{i=1}^{n} \mathrm{Cl}_\lambda(\mathrm{U}_{\lambda_i})$ implying that X is C-λ-compact.

Theorem 2.3: A λ-continuous image of a C-λ-compact space is C-λ-compact.

Proof: Let A be a closed subset of Y and let V be an λ-open cover of A. By λ-continuity of f, $f^{-1}(\mathrm{A})$ is an λ-closed subset of X and is such that $\mathrm{P} = \{f^{-1}(\mathrm{V}) : \mathrm{V} \in \mathcal{V}\}$ is a cover of $f^{-1}(\mathrm{A})$ by λ-open sets. By C-λ-compactness of X, there exists finite collection say; $\{\mathrm{P}_i : 1 \le i \le n\}$ of P such that $f^{-1}(\mathrm{A}) \subset \bigcup_{\mathrm{i}=1}^{n} \{\mathrm{Cl}_\lambda(f^{-1}(\mathrm{V}_i)) : 1 \le i \le n\}$. Now by λ-continuity of f, $\mathrm{A} \subset \bigcup_{\mathrm{i}=1}^{n} \{\mathrm{Cl}_\lambda(\mathrm{V}_i) : 1 \le i \le n\}$. Thus Y is a C-$\lambda$-compact space.

Definition 2.4: Let X be a topological space and A be a subset of X then an element $x \in \mathrm{X}$ is called λ-adherent point of A if every λ-open set G containing *x* contains at least one point of A, that is, $\mathrm{G} \cap \mathrm{A} \neq \phi$.

Definition 2.5: Let X be a non-empty set. A non-empty collection B of non-empty subset of X is called a basis for some filter on X if

(i) $\phi \notin \mathcal{B}$

(ii) If $\mathrm{B}_1, \mathrm{B}_2 \in \mathcal{B}$ then there exist a $\mathrm{B} \in \mathcal{B}$ such that $\mathrm{B} \subset \mathrm{B}_1 \cap \mathrm{B}_2$.

Definition 2.6: Let $\mathcal{B}$ be a filter base on a set X then the filter $\mathcal{F}$ is said to be generated by $\mathcal{B}$ if $\mathcal{F} = \{\mathrm{A} : \mathrm{A} \subset, \mathrm{B} \in \mathcal{B}\}$

Definition 2.7: A filter base $\mathcal{F}$ is said to be λ-adherent convergent if every neighborhood of the λ-adherent set of $\mathcal{F}$ contains an element of $\mathcal{F}$.

Theorem 2.8: For any λ-Hausdorff space X. The following properties are equivalent.

(i) X is C-λ-compact.

(ii) For each closed subset $A \subset X$ and for each family f of closed set of X with $\cap\{F \cap A : F \in f\} = \phi$ there exists finite collection say; $\{F_i : 1 \le i \le n\}$ of f with $\bigcap_{i=1}^{n}\{(\text{int}_\lambda F_i) \cap A : 1 \le i \le n\} = \phi$.

(iii) If A is a closed subset of X and f is an open filter base on X whose element have non-empty with A, then f has a λ-adherent point in X.

Proof: (i)⇒(ii): Let A be a closed subset of a C-λ-compact space X and f be a family of closed subset of X with $\cap\{F \cap A : F \in f\} = \phi \Rightarrow A \subset X \sim \cap\{F : F \in f\}$ or $A \subset \cup\{X \sim F : F \in f\}$. Therefore $\{X \sim F : F \in f\}$ is an open cover of A. Since every open set is a λ-open set, therefore $\mathcal{G} = \{X \sim F : F \in f\}$ is a λ--open cover of A and so by C-λ-compactness of X , there exists a finite subfamily say; $\{X \sim F_i : 1 \le i \le n \text{ and } F_i \in f\}$ of $\mathcal{G}$ such that $A \subset \bigcup_{i=1}^{n}\{\text{Cl}_\lambda(X \sim F_i) : 1 \le i \le n\} \Rightarrow A \subset X \sim \bigcap_{i=1}^{n}\{(\text{Int}_\lambda(F_i)) : 1 \le i \le n\}$. Therefore $\bigcap_{i=1}^{n}\{(\text{int}_\lambda F_i) \cap A : 1 \le i \le n\} = \phi$.

(ii)⇒(iii): Suppose that there exist a closed set A and let $\mathcal{G}$ be an open filter base having non-empty trace with A such that $\mathcal{G}$ has no λ-adherent point. Now $f = \{\text{Cl}_\lambda(G) : G \in \mathcal{G}\}$ is a family of closed set such that $\cap\{\text{Cl}_\lambda(G) : G \in \mathcal{G}\} \cap A = \phi$. But $\text{Cl}_\lambda(G)$ is a superset of G so $\text{Cl}_\lambda(G) \in \mathcal{G}$. Therefore $\cap\{(\text{Cl}_\lambda(G)) \cap A : G \in \mathcal{G}\}$ is a trace of A and by hypothesis $\mathcal{G}$ has no λ-adherent point, therefore $\cap\{(\text{Cl}_\lambda(G)) \cap A : G \in \mathcal{G}\} = \phi$] so there is a finite subfamily of f, say $\{F_i = \text{Cl}_\lambda(G_i) : 1 \le i \le n\}$ with $\bigcap_{i=1}^{n}\{(\text{int}_\lambda F_i) \cap A : 1 \le i \le n\} = \phi$ or $A \subset \bigcup_{i=1}^{n}\{X \sim \text{int}_\lambda(\text{Cl}_\lambda(G_i)) : 1 \le i \le n\}$. Therefore $\bigcap_{i=1}^{n}\{G_i \cap A : 1 \le i \le n\} = \phi$. Since $\mathcal{G}$ is a filter base, therefore there must exist a $G \in \mathcal{G}$ such that $G \subset \bigcap_{i=1}^{n}\{G_i : 1 \le i \le n\}$. So $G \cap A = \phi$, a contradiction.

(iii)⇒(i): Assume that X is not C-λ-compact then there is a closed subset A and a covering $\mathcal{U}$ of A consisting of λ-open subset of X such that for any finite subfamily $\{U_i : 1 \le i \le n\}$ of $\mathcal{U}$, $A \not\subset \bigcup_{i=1}^{n}\{\text{Cl}_\lambda(U_i) : 1 \le i \le n\}$. Now $\mathcal{G} = \left[X \sim \bigcup_{i=1}^{n}\{\text{Cl}_\lambda(U_i) : 1 \le i \le n \text{ and } U_i \in \mathcal{U}\}\right]$ is an open filter base having non-empty trace with A, so by (iii) there is an λ -adherent point of G in A, let it be *x*. Thus $x \in \mathrm{Cl}_\lambda\left[\mathrm{X} \sim \bigcup_{i=1}^{n}\{\mathrm{Cl}_\lambda(\mathrm{U}_i) : 1 \le i \le n\}\right]$ say; for each $\mathcal{G} \in$ G or $x \in \left[\mathrm{X} \sim \bigcup_{i=1}^{n}\{\mathrm{U}_i : 1 \le i \le n\}\right]$ Therefore, $\mathcal{U}$ is not a covering of A, a contradiction.

Theorem 2.9: A λ-Hausdorff space X is C-λ-compact if and only if every open filter base $\mathcal{F}$ is λ-adherent convergent.

Proof: Let $\mathcal{F}$ be an open filter base of the C-λ-compact space X. Let A be λ-adherent set of $\mathcal{F}$. Let G be an open-neighborhood of A. Since A is the λ-adherent set of $\mathcal{F}$, we have $\mathrm{A} = \cap\{\mathrm{Cl}_\lambda(\mathrm{F}) : \mathrm{F} \in \mathcal{F}\}$. Since G is an open-neighborhood of A, we have $\mathrm{A} \subset \mathrm{G}$ and $\mathrm{X} \sim \mathrm{G}$ is closed. Clearly $\{\mathrm{X} \sim \mathrm{Cl}_\lambda(\mathrm{F}) : \mathrm{F} \in \mathcal{F}\}$ is an λ-open cover of $\mathrm{X} \sim \mathrm{G}$ and so $\mathrm{X} \sim \mathrm{G} \subset \bigcup_{i=1}^{n}\{\mathrm{Cl}_\lambda(\mathrm{X} \sim \mathrm{Cl}_\lambda \mathrm{F}_i) : 1 \le i \le n\}$. This implies $\bigcap_{i=1}^{n}\{\mathrm{Cl}_\lambda(\mathrm{X} \sim \mathrm{Cl}_\lambda \mathrm{F}_i) : 1 \le i \le n\} \subset \mathrm{G}$. Further $\mathrm{X} \sim \mathrm{Cl}_\lambda \mathrm{F}_i \subset \mathrm{X} \sim \mathrm{F}_i$ or $\bigcap_{i=1}^{n}\{\mathrm{F}_i : 1 \le i \le n\} \subset \bigcap_{i=1}^{n}\{\mathrm{Cl}_\lambda(\mathrm{X} \sim \mathrm{Cl}_\lambda \mathrm{F}_i) : 1 \le i \le n\}$. Thus $\bigcap_{i=1}^{n}\{\mathrm{F}_i : 1 \le i \le n\} \subset \mathrm{G}$, that is, open-neighborhood G of A contains a point of $\mathcal{F}$.

Conversely, let X be a non C-λ-compact space and let A be any closed subset of X. Choose an λ-open cover $\mathcal{U}$ of A such that A is not contained in the λ-closure of any finite union of elements in $\mathcal{U}$. Without loss of generality we may consider $\mathcal{U}$ to be closed under finite unions. Obviously, then $\mathcal{F} = \{\mathrm{X} \sim \mathrm{Cl}_\lambda \mathrm{G} : \mathrm{G} \in \mathcal{U}\}$ an open filter base in X. Let *x* be an λ-adherent point of $\mathcal{F}$. This clearly implies that $x \notin \mathrm{A}$. So the λ-adherent set of the open filter base $\mathcal{F}$ is contained in $\mathrm{X} \sim \mathrm{A}$, but no element of $\mathcal{F}$ is contained in $\mathrm{X} \sim \mathrm{A}$.

Theorem 2.10: A λ-Hausdorff space X is C-λ-compact if and only if for each closed subset C of X and λ open cover C of $\mathrm{X} \sim \mathrm{C}$ and a open-neighborhood U of C, there exists a finite collection $\{\mathrm{G}_i \in C : 1 \le i \le n\}$ such that $\mathrm{X} = \mathrm{U} \cup \bigcup_{i=1}^{n}\{\mathrm{Cl}_\lambda \mathrm{G}_i : 1 \le i \le n\}$.

Proof: Since U is an open-neighborhood of C, therefore $\mathrm{C} \subset \mathrm{U} \subset \mathrm{Cl(C)}$, or $\mathrm{X} \sim \mathrm{U} \subset \mathrm{X} \sim \mathrm{C}$ where $\mathrm{X} \sim \mathrm{U}$ is a λ-closed set. Further, as C is a λ-open cover of $\mathrm{X} \sim \mathrm{C}$. Therefore C is a λ-

open cover of the α-closed set $X \sim U$ too. Now by C-λ-compactness of X, there exists a finite subfamily $\{G_i : 1 \le i \le n\}$ of C such that $X \sim U \subset \bigcup_{i=1}^{n} \{Cl_\lambda G_i : 1 \le i \le n\}$ which implies

$X = U \cup \bigcup_{i=1}^{n} \{Cl_\lambda G_i : 1 \le i \le n\}$.

Conversely; Let A be a closed subset of X, G be an λ-open cover of A, Therefore $A \subset \bigcup \{G : G \in \mathcal{G}\} = H$ (say), obviously H is λ-open, therefore $X \sim H = C$ (say), is λ-closed and $C \subset X \sim A$. Since $X \sim A$ is λ-open. Therefore we can take $X \sim A = U$ is an open neighborhood of C, thus by the given statement $X = U \cup \bigcup_{i=1}^{n} \{Cl_\lambda G_i : 1 \le i \le n\}$. Hence X is C-$\lambda$-compact.

Theorem 2.11: Every λ-continuous function from a C-λ-compact space to a λ-Hausdorff space is closed.

Proof: Let f be λ-continuous function from a C-λ-compact space X to a λ-Hausdorff space Y. Let C be a closed set in X and let $P \notin f(C)$. Now for every $x \in f(C), x \ne p$ and hence choose a open-neighborhood N_x such that $P \notin Cl_\lambda(N_x)$, obviously $\{f^{-1}(N_x) : x \in f(C)\}$ is a λ-open cover of C. Let $\{x_i : 1 \le i \le n\}$ be such that, $C \subset \bigcup_{i=1}^{n} \{Cl_\lambda f^{-1}(N_{x_i}) : 1 \le i \le n\}$ because X is C-λ-compact space. Thus by the λ-continuity of f, $Y \sim \bigcup_{i=1}^{n} \{Cl_\lambda(N_{x_i}) : 1 \le i \le n\}$ λ-neighborhood of p disjoint from Y . Hence C is closed, so λ-continuous function f from C-λ-compact space X to a λ-Hausdorff space Y is closed.

Definition 2.12: A Hausdorff space X is said to be a functionally compact space if for every open filter base U in X, the intersection A of the elements of U is equal to the intersection of the closure of the elements of U, then U is a base for the neighbourhood of A.

Theorem 2.13: Every C-λ-compact space is functionally compact.

Proof: Let $\mathcal{U}$ be an λ-open filter base in the C-λ-compact space X. Let $A = \cap(U : U \in \mathcal{U})$ $= \bigcap(Cl_\lambda U : U \in \mathcal{U})$. Let G be an λ-open set containing A. Then $\cap(Cl_\lambda U : U \in \mathcal{U})$ is a subset of G, that is, $\cap\{X \sim Cl_\lambda(U) : U \in \mathcal{U}\}$. Now $X \sim G$ is a λ-closed subset of the C-λ-compact space X. Therefore the λ-open cover $(X \sim Cl_\lambda(U) : U \in \mathcal{U})$ of $X \sim G$ has a finite subfamily, say $(X \sim Cl_\lambda(U_i) : 1 \le i \le n)$ such that $X \sim G \subset \cup(Cl_\lambda(X \sim Cl_\lambda(U_i)) : 1 \le i \le n)$ $\subset (X \sim Cl_\lambda(U_i) : 1 \le i \le n)$, that is, $\cap\{U_i : 1 \le i \le n\} \subset G$. Since U is a filter base there exists a $U \in \mathcal{U}$ such that $U \subset \bigcap\{U_i : 1 \le i \le n\}$ and hence $U \subset G$ and the space X is functionally compact.

Theorem 2.14: A λ-Hausdorff space X is functionally compact if and only if for every λ-regular closed subset C of X and λ-open cover B of $X \sim C$ and a open-neighborhood U of C, there exists a finite collection $\{B_{x_i} \in \mathcal{B} : 1 \le i \le n\}$ such that

$$X = U \cup \left[\bigcup_{i=1}^{n}\{Cl_\lambda(B_{x_i}) : 1 \le i \le n\}\right].$$

Proof: For each $x \in X \sim C$, since C is λ-regular closed, there exists an λ-open set A_x such that $Cl_\lambda A_x \subset X \sim C$. Also there exists a $B_x \in \mathcal{B}$ such that $x \in B_x$. Let $G_x = A_x \bigcap B_x$. Then G_x is an λ-open set such that $x \in G_x$, $Cl_\lambda G_x \subset X \sim C$ and there exists a $B_x \in \mathcal{B}$ such that $G_x \subset B_x$. Also $X \sim C = \cup\{G_x : x \in X \sim C\} = \cup\{Int_\lambda Cl_\lambda G_x : x \in X \sim C\} = \cup\{Cl_\lambda G_x : x \in X \sim C\}$. Suppose, if possible, that no finite collection of G is such that the λ-closure of its members cover $X \sim U$. Now for any finite collection $\{G_{x_i} : 1 \le i \le n\}$ of G, $\cap\{X \sim Cl_\lambda G_{x_i} : 1 \le i \le n\} \ne \phi$. Let V be the family of the all finite intersection of the family $\{X \sim Cl_\lambda G_x : x \in X \sim C\}$. Now V is an λ -open filter base such that $\cap\{V : V \in \mathcal{V}\} = \cap\{X \sim Cl_\lambda G_x : x \in X \sim C\}$ $= X \sim \cap\{Cl_\lambda G_x : x \in X \sim C\} = C$ and $\cap\{Cl_\lambda V : V \in \mathcal{V}\} = \cap\{Cl_\lambda(X \sim Cl_\lambda G_x) : x \in X \sim C\}$ $= \cap\{(X \sim Int_\lambda Cl_\lambda G_x) : x \in X \sim C\} = X \sim \cup\{(Int_\lambda Cl_\lambda G_x) : x \in X \sim C\} = C$. But there exists no $V \in \mathcal{V}$ such that $C \subset U \subset V$ and this is a contradiction to the fact that X is functionally compact. Hence there exists a finite collection $\{G_{x_i} : 1 \le i \le n\}$ and hence $\{B_{x_i} : 1 \le i \le n\}$ such that

$$X = U \cup \left[\bigcup_{i=1}^{n}\{Cl_\lambda(B_{x_i}) : 1 \le i \le n\}\right].$$

Conversely; Let U be an λ-open filter base such that $A = \cap(U : U \in \mathcal{U}) = \cap(Cl_\lambda U : U \in \mathcal{U})$. Let G be an λ-open set containing A. Now for each $x \in X \sim A$, there exists a $U \in \mathcal{U}$ such that $x \notin Cl_\lambda(U)$. Now $x \in X \sim Cl_\lambda(U)$ and $Cl_\lambda(X \sim Cl_\lambda(U)) \cap A = \phi$, because $A \subset U$ for each $U \in \mathcal{U}$. Therefore, A is a λ -regular closed set. $\{X \sim Cl_\lambda(U) : U \in \mathcal{U}\}$ is an λ -open cover of $X \sim A$. Therefore there exists a finite collection $\{X \sim Cl_\lambda(U_i) : 1 \le i \le n\}$ of $\{X \sim Cl_\lambda(U) : U \in \mathcal{U}\}$ such that $X = G \cup [\cup \{Cl_\lambda(X \sim Cl_\lambda(U_i)) : 1 \le i \le n\}]$. Thus $X \sim G \subset \cup \{Cl_\lambda(X \sim Cl_\lambda(U_i)) : 1 \le i \le n\}$ that is, $\cup \{Int_\lambda Cl_\lambda(U_i) : 1 \le i \le n\} \subset G$ that is, $\cup \{U_i : 1 \le i \le n\} \subset G$. Since U is an λ -open filter base, there exists a $U \in \mathcal{U}$ such that $U \subset \{U_i : 1 \le i \le n\}$ and hence $U \subset G$. Thus X is functionally compact.

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