Ratio ranking method of Triangular Single valued Neutrosophic numbers

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Abstract: The concept of a single valued neutrosophic number (SVN-number) is of importance for quantifying an ill-known quantity and the ranking of SVN-numbers is a very difficult problem. Ranking of single valued neutrosophic numbers is essential for handling optimization, multi-attribute decision making, logistic and supply chain problems etc. The aim of this article is to provide a methodology for ranking of triangular single valued Neutrosophic numbers. Initially the values and ambiguities of truth membership function, indeterminacy and the falsity membership function for a triangular single valued Neutrosophic number are defined. Then a new ordering procedure is developed on the basis of a ratio of the value index to the ambiguity index. Finally the proposed ordering method is illustrated by numerical examples.

Keywords: Neutrosophic fuzzy set, Neutrosophic Trapezoidal fuzzy number, Neutrosophic Triangular fuzzy number, Value of Triangular Neutrosophic number, Ambiguity of a Triangular Neutrosophic number and Ratio ranking of triangular neutrosophic number.

1 Introduction

The intuitionistic fuzzy set (IFS) by Attanassov [15] by adding an additional non-membership degree, which may express more abundant and flexible information as compared with the fuzzy set was an extension of the fuzzy set [16] introduced by Zadeh. For the past few years, a quite a large number of ranking methods have been developed to many kind of intuitionistic fuzzy numbers [14,17, 22]. The concept of neutrosophic set by Smarandache [2,3] can provide a generalization of fuzzy set and intuitionistic fuzzy set that make it is the best fit in representing indeterminacy and uncertainty. Neutrosophic theory involves philosophy viewpoint which addresses nature and scope of neutralities, as well as their interactions with different ideational spectra. Triangular Single Valued Triangular Neutrosophic Numbers (TrSVNN) is a special case of neutrosophic set that can handle ill-known quantity very difficult problems. Different types of linear and non-linear generalized triangular neutrosophic numbers were defined and the de-neutrosophication concept for neutrosophic number for triangular neutrosophic numbers was introduced to handle imprecise project evaluation review technique and route selection problem by Chakraborty et.al.[1].Wang et al. [4] applied SV-trapezoidal neutrosophic preference in decision making problem. Deli et.al [5] developed a ranking method by using the concept of values and ambiguities, and applied to multi-attribute decision making problems in which the ratings of alternatives on attributes are expressed with SVTN-numbers. Single valued trapezoidal neutrosophic operators were proposed by Deli [6, 9] and applied them in a decision making problem. The same author [7], developed linear optimization method of SVN-sets to describe the sensitivity analysis of attribute weights which give changing intervals of attribute weights in which the ranking order of the alternatives is required to remain unchanging. Deli and

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Subas [8] designed weighted geometric operators under single valued triangular neutrosophic numbers to solve decision making problem. Deli et al.[10] solved a decision making problem using neutrosophic soft sets. Ye [12,13] introduced the notations of simplified neutrosophic sets and developed a ranking method. Then, he introduced some aggregation operators. Basset et al. [18] proposed a hybrid method of neutrosophic sets and method of DEMATEL to develop criteria for supplier selection. Basset et al. [19, 20] proposed a structure based on VIKOR technique for e-government website evaluation. Basset et al. [21] proposed a new method for a neutrosophic linear programming problem. Generalized single valued triangular neutrosophic number was defined and a new ranking based on new score functions by applying the Hamming distance was proposed. Then it was applied to obtain solution for multi-attribute group decision making problems by Mehmet et.al in 2018 [23]. Biswas et al. [24] proposed distance measure using interval trapezoidal neutrosophic numbers. Biswas et al. [25] developed a ranking method based on value and ambiguity index based on single-valued trapezoidal neutrosophic numbers. Broumi et al. [26] proposed some notion with respect to neutrosophic set with triangular and trapezoidal concept and primary operations as well. Also done a contingent analysis with the existing concepts and proposed neutrosophic numbers. Broumi et.al. [27], a new score function is proposed for interval valued neutrosophic numbers and SPP is solved using interval valued neutrosophic numbers. The same authors [28] proposed an algorithm method for finding the shortest path length between paired nodes on a network where the edge weights are characterized by single valued triangular neutrosophic numbers. Ibrahim et.al. [29] developed a ranking method by using the concept of values and ambiguities and applied it to evaluate information system quality.

Structure of the paper: Some underlying basic concepts and definition needed for the present research has been provided in section 2. Section 3 gives some definitions related to cut sets, value and ambiguity of truth, indeterminacy and falsity membership functions. Then value and ambiguity indices were defined and few properties related to these indices were proved. Finally a ratio of value to the ambiguity index was defined and using which new ranking procedure was proposed. Some numerical examples with pictorial representation were illustrated to explain the designed ranking of TrSVNNs in Section 4. The last section furnishes the conclusion of research work done and scope for future research.

2 PRELIMINARIES

This section gives a brief outline of various fundamental definitions related to neutrosophic sets and neutrosophic numbers and basic arithmetic operations on triangular single valued neutrosophic numbers.

Definition 2.1 [3] An IFS A in X is given by $A=\{(x,\mu_A(x),\nu_A(x),x\in X)\}$, where the functions $\mu_A(x):X\rightarrow[0,1]$ and $\nu_A(x):X\rightarrow[0,1]$ define respectively, the degree of membership and degree of non-membership of the element $x\in X$ to the set A, which is a subset of X, and for every $x\in X, 0\le\mu_A(x)+\nu_A(x)\le 1$. Obviously, every fuzzy set has the form $\{(x,\mu_A(x),\mu_{A^c}(x)),x\in X\}$.

For each IFS A in X, we will call $\Pi_A(x)=1-\mu(x)-\nu(x)$ the intuitionistic fuzzy index of x in A. It is obvious that $0\le\Pi_A(x)\le 1$, for all $x\in X$.

Definition 2.2 [12] Let X be a universe of discourse, then a neutrosophic set $\tilde{N}$ in X is given by

$$\tilde{N} = \left\{ \left\langle x, T_{\tilde{N}}(x), I_{\tilde{N}}(x), F_{\tilde{N}}(x) \right\rangle \middle| x \in X \right\},$$

where $T_{\tilde{N}}(x), I_{\tilde{N}}(x)$ and $\mathrm{F}_{\tilde{N}}(x)$ are called truth-membership function, indeterminacy-membership function and falsity membership function, respectively. They are respectively defined by $T_{\tilde{N}}(x) \subset \left]^{-}0,1^{+}\right[, I_{\tilde{N}}(x) \subset \left]^{-}0,1^{+}\right[$, and $\mathrm{F}_{\tilde{N}}(x) \subset \left]^{-}0,1^{+}\right[$ such that $0^{-} \le T_{\tilde{N}}(x) + I_{\tilde{N}}(x) + \mathrm{F}_{\tilde{N}}(x) \le 3^{+}$.

Definition 2.3[1] A Triangular Single Valued Neutrosophic number(TrSVNN) of Type 1 is defined as $\tilde{A} = (p_1, p_2, p_3; q_1, q_2, q_3; r_1, r_2, r_3)$, whose truth membership, indeterminacy and falsity membership is defined as follows:

$$T_{\tilde{A}}(x) = \begin{cases} \dfrac{x - p_1}{p_2 - p_1}, \text{when } p_1 \le x < p_2 \\ 1, \text{when } x = p_2 \\ \dfrac{p_3 - x}{p_3 - p_2}, \text{when } p_2 < x \le p_3 \\ 0, \text{otherwise} \end{cases}$$

$$I_{\tilde{A}}(x) = \begin{cases} \dfrac{q_2 - x}{q_2 - q_1}, \text{when } q_1 \le x < q_2 \\ 0, \text{when } x = q_2 \\ \dfrac{x - q_2}{q_3 - q_2}, \text{when } q_2 < x \le q_3 \\ 1, \text{otherwise} \end{cases}$$

$$F_{\tilde{A}}(x) = \begin{cases} \dfrac{r_2 - x}{r_2 - r_1}, \text{when } r_1 \le x < r_2 \\ 0, \text{when } x = r_2 \\ \dfrac{x - r_2}{r_3 - r_2}, \text{when } r_2 < x \le r_3 \\ 1, \text{otherwise} \end{cases}$$

3 Ranking of Neutrosophic Triangular Fuzzy Numbers by ratio of value and ambiguity indices

In this section, we first define the concept of cut (or level) sets, values, ambiguities, value index and ambiguity indices of TrSVNNs and proved some desired properties. Finally we developed a ranking method for ordering TrSVNNs by means of ratio of value and ambiguity indices.

Definition 3.1: Let $\tilde{A} = (p_1, p_2, p_3; q_1, q_2, q_3; r_1, r_2, r_3)$, be a TrSVNN. Then $\alpha - cut\,set$ of $\tilde{A}$ is a crisp subset of $\mathbb{R}$ denoted by $\tilde{A}_\alpha, 0 \le \alpha \le 1$ is the closed interval defined by $\tilde{A}_\alpha = \{x / T_{\tilde{A}}(x) \ge \alpha\} = \left[L_{\tilde{A}}(\alpha), R_{\tilde{A}}(\alpha)\right]$. Using the definition and parametric form we obtain

$\tilde{A}_\alpha = \left[p_1 + \alpha\left(p_2 - p_1\right), p_3 - \alpha\left(p_3 - p_2\right) \right].$

Definition 3.2: Let $\tilde{A} = \left(p_1, p_2, p_3; q_1, q_2, q_3; r_1, r_2, r_3\right)$, be a TrSVNN. Then $\beta - cut\,set$ of $\tilde{A}$ is a crisp subset of $\mathbb{R}$ denoted by $\tilde{A}_\beta, 0 \le \beta \le 1$ is the closed interval defined by $\tilde{A}_\beta = \left\{ x / I_{\tilde{A}}\left(x\right) \le \beta \right\} = \left[L_{\tilde{A}}\left(\beta\right), R_{\tilde{A}}\left(\beta\right) \right]$. Using the definition and parametric form we obtain $\tilde{A}_\beta = \left[q_2 - \beta\left(q_2 - q_1\right), q_2 + \beta\left(q_3 - q_2\right) \right]$.

Definition 3.3: Let $\tilde{A} = \left(p_1, p_2, p_3; q_1, q_2, q_3; r_1, r_2, r_3\right)$, be a TrSVNN. Then $\gamma - cut\,set$ of $\tilde{A}$ is a crisp subset of $\mathbb{R}$ denoted by $\tilde{A}_\gamma, 0 \le \gamma \le 1$ is the closed interval defined by $\tilde{A}_\gamma = \left\{ x / F_{\tilde{A}}\left(x\right) \le \gamma \right\} = \left[L_{\tilde{A}}\left(\gamma\right), R_{\tilde{A}}\left(\gamma\right) \right]$. Using the definition and parametric form we obtain $\tilde{A}_\gamma = \left[r_2 - \gamma\left(r_2 - r_1\right), r_2 + \gamma\left(r_3 - r_2\right) \right]$.

3.1. Value and ambiguity of a TrSVNN

Definition 3.4: Let $\tilde{A}_\alpha, \tilde{A}_\beta, \tilde{A}_\gamma$ be the $\alpha - cut,\ \beta - cut,\ and\ \gamma - cut$ sets of a TrSVNN $\tilde{A}$. Then the value of truth membership function $T_{\tilde{A}}\left(x\right)$, indeterminacy function $I_{\tilde{A}}\left(x\right)$, and falsity membership function $F_{\tilde{A}}\left(x\right)$ of TrSVNN $\tilde{A}$ are respectively defined by

$$V_T\left(\tilde{A}\right) = \int_0^1 \left(L_{\tilde{A}}\left(\alpha\right) + R_{\tilde{A}}\left(\alpha\right) \right) f\left(\alpha\right) d\alpha \qquad (1)$$

$$V_I\left(\tilde{A}\right) = \int_0^1 \left(L_{\tilde{A}}\left(\beta\right) + R_{\tilde{A}}\left(\beta\right) \right) g\left(\beta\right) d\beta \qquad (2)$$

$$V_F\left(\tilde{A}\right) = \int_0^1 \left(L_{\tilde{A}}\left(\gamma\right) + R_{\tilde{A}}\left(\gamma\right) \right) h\left(\gamma\right) d\gamma \qquad (3)$$

Here are weighing functions which are non-negative and non-decreasing, that can be set depending upon the nature decision making problems by the decision maker in the interval [0,1]. The function $f\left(\alpha\right)$ gives different weights to elements at different α - cut sets so it can lessen the contribution of the lower α - cut sets, since these cut sets arising from values of truth membership function $T_{\tilde{A}}\left(x\right)$ have a considerable amount of uncertainty. Therefore, $V_T\left(\tilde{A}\right)$ synthetically reflects the information on truth membership degrees. The function $g\left(\beta\right)$ can lessen the contribution of the higher β - cut sets since these cut sets, arising from values of indeterminacy function $I_{\tilde{A}}\left(x\right)$, have a considerable amount of uncertainty. $V_I\left(\tilde{A}\right)$ synthetically reflects the information on non-membership degrees. Likewise $h\left(\gamma\right)$ has the effect of weighting on the different γ - cut sets arising from the values of falsity membership function $F_{\tilde{A}}(x)$, have a considerable amount of uncertainty. $V_F\left(\tilde{A}\right)$ synthetically reflects the information on every falsity degree and may be regarded as a central value that represents from the falsity membership function point of view.

Without loss of generality, for deriving the value and ambiguity indices, let us assume $f(\alpha)=\alpha, g(\beta)=1-\beta$ and $\mathrm{h}(\gamma)=1-\gamma,\ \alpha,\beta,\gamma\in[0,1]$. The values of truth membership function $T_{\tilde{A}}(x)$, indeterminacy function $I_{\tilde{A}}(x)$, and falsity membership function $F_{\tilde{A}}(x)$ of TrSVNN $\tilde{A}$ are derived as follows:

$$V_T\left(\tilde{A}\right)=\int_0^1\left[L_{\tilde{A}}(\alpha)+R_{\tilde{A}}(\alpha)\right]f(\alpha)d\alpha$$

$$=\int_0^1\left[L_{\tilde{A}}(\alpha)+R_{\tilde{A}}(\alpha)\right].\alpha\ d\alpha$$

$$=\int_0^1\left[\left\{p_1+\alpha\left(p_2-p_1\right)\right\}+\left\{p_3-\alpha\left(p_3-p_2\right)\right\}\right].\alpha\ d\alpha$$

$$=\int_0^1\left[p_1\alpha+\alpha^2\left(p_2-p_1\right)+p_3\alpha-\left(p_3-p_2\right)\alpha^2\right]d\alpha$$

$$=\left[\left(p_1+p_3\right)\frac{\alpha^2}{2}+\left(2p_2-p_1-p_3\right)\frac{\alpha^3}{3}\right]_0^1=\frac{p_1+p_3}{2}+\frac{\left(2p_2-p_1-p_3\right)}{3}$$

Hence $V_T\left(\tilde{A}\right)=\dfrac{p_1+4p_2+p_3}{6}$ (4)

$$V_I\left(\tilde{A}\right)=\int_0^1\left[L_{\tilde{A}}(\beta)+R_{\tilde{A}}(\beta)\right]g(\beta)d\beta$$

$$=\int_0^1\left[\left\{q_2-\beta\left(q_2-q_1\right)\right\}+\left\{q_2+\beta\left(q_3-q_2\right)\right\}\right].(1-\beta)\ d\beta$$

$$=\int_0^1\left[2q_2+\beta\left(q_1-4q_2+q_3\right)+\beta^2\left(2q_2-q_1-q_3\right)\right]d\alpha$$

$$=\frac{q_1+4q_2+q_3}{6}$$

Hence $V_I\left(\tilde{A}\right)=\dfrac{q_1+4q_2+q_3}{6}$ (5)

$$V_F\left(\tilde{A}\right)=\int_0^1\left[L_{\tilde{A}}(\gamma)+R_{\tilde{A}}(\gamma)\right]h(\gamma)d\gamma$$

$$=\int_0^1\left[L_{\tilde{A}}(\gamma)+R_{\tilde{A}}(\gamma)\right].(1-\gamma)\ d\gamma$$

$$=\int_0^1\left[\left\{r_2-\gamma\left(r_2-r_1\right)\right\}+\left\{r_2+\gamma\left(r_3-r_2\right)\right\}\right].(1-\gamma)\ d\gamma$$

$$=\int_0^1\left[2r_2+\gamma\left(r_1-4r_2+r_3\right)+\gamma^2\left(2r_2-r_1-r_3\right)\right]d\alpha=\frac{r_1+4r_2+r_3}{6}$$

Hence $V_T\left(\tilde{A}\right)=\dfrac{r_1+4r_2+r_3}{6}$ (6)

Definition 3.5: Let $\tilde{A}_\alpha, \tilde{A}_\beta, \tilde{A}_\gamma$ be the $\alpha-cut,\ \beta-cut,\ and\ \gamma-cut$ sets of a TrSVNN $\tilde{A}$. Then the value of truth membership function $T_{\tilde{A}}(x)$, indeterminacy function $I_{\tilde{A}}(x)$, and falsity membership function $F_{\tilde{A}}(x)$ of TrSVNN $\tilde{A}$ are respectively defined by

$$A_T\left(\tilde{A}\right)=\int_0^1\left(R_{\tilde{A}}(\alpha)-L_{\tilde{A}}(\alpha)\right)f(\alpha)\,d\alpha \tag{7}$$

$$A_I\left(\tilde{A}\right)=\int_0^1\left(R_{\tilde{A}}(\beta)-L_{\tilde{A}}(\beta)\right)g(\beta)\,d\beta \tag{8}$$

$$A_F\left(\tilde{A}\right)=\int_0^1\left(R_{\tilde{A}}(\gamma)-L_{\tilde{A}}(\gamma)\right)h(\gamma)\,d\gamma \tag{9}$$

It is easily seen that $R_{\tilde{A}}(\alpha)-L_{\tilde{A}}(\alpha), R_{\tilde{A}}(\beta)-L_{\tilde{A}}(\beta)$ and $R_{\tilde{A}}(\gamma)-L_{\tilde{A}}(\gamma)$ are just about the lengths of the intervals $\tilde{A}_a, \tilde{A}_\beta$ and $\tilde{A}_\gamma$ respectively. Thus, $A_T\left(\tilde{A}\right), A_I\left(\tilde{A}\right)$ and $A_F\left(\tilde{A}\right)$ can be regarded as the global spreads of the truth, indeterminacy, and falsity membership function respectively. The ambiguity of truth, indeterminacy, and falsity membership functions are derived as follows:

$$\begin{aligned}
A_T\left(\tilde{A}\right)&=\int_0^1\left[R_{\tilde{A}}(a)-L_{\tilde{A}}(\alpha)\right]f(\alpha)\,d\alpha\\
&=\int_0^1\left[\left\{p_3-\alpha\left(p_3-p_2\right)\right\}-\left\{p_1+\alpha\left(p_2-p_1\right)\right\}\right].\alpha\ d\alpha\\
&=\int_0^1\left[\left(p_3-p_1\right)\alpha-\left(2p_2+p_1+p_3\right)\alpha^2\right]d\alpha\\
&=\left[\left(p_3-p_1\right)\frac{\alpha^2}{2}-\left(p_3-p_1\right)\frac{\alpha^3}{3}\right]_0^1=\frac{p_3-p_1}{6}
\end{aligned}$$

Hence $A_T\left(\tilde{A}\right)=\dfrac{p_3-p_1}{6}$ (10)

$$\begin{aligned}
A_I\left(\tilde{A}\right)&=\int_0^1\left[R_{\tilde{A}}(\beta)-L_{\tilde{A}}(\beta)\right]g(\beta)\,d\beta\\
&=\int_0^1\left[\left\{q_2+\beta\left(q_3-q_2\right)\right\}-\left\{q_2-\beta\left(q_2-q_1\right)\right\}\right].(1-\beta)\ d\beta\\
&=\frac{q_3-q_1}{6}
\end{aligned}$$

Hence $A_I\left(\tilde{A}\right)=\dfrac{q_3-q_1}{6}$ (11)

$$A_F\left(\tilde{A}\right)=\int_0^1\left[R_{\tilde{A}}(\gamma)-L_{\tilde{A}}(\gamma)\right]h(\gamma)d\gamma$$
$$=\int_0^1\left[\left\{r_2+\gamma\left(r_3-r_2\right)\right\}-\left\{r_2-\gamma\left(r_2-r_1\right)\right\}\right].(1-\gamma)\ d\gamma$$
$$=\frac{r_3-r_1}{6}$$

Hence $A_F\left(\tilde{A}\right)=\frac{r_3-r_1}{6}$ (12)

3.2 Properties related to value and ambiguity

Theorem 3.1: Let $\tilde{A}=\left(p_1,p_2,p_3;q_1,q_2,q_3;r_1,r_2,r_3\right)$ and $\tilde{B}=\left(t_1,t_2,t_3;u_1,u_2,u_3;v_1,v_2,v_3\right)$ be any two TrSVNNs. Then the equation $V_T\left(\tilde{A}+\tilde{B}\right)=V_T\left(\tilde{A}\right)+V_T\left(\tilde{B}\right)$ is valid.

Proof: From definition, we have
$\tilde{A}+\tilde{B}=\left(p_1+t_1,p_2+t_2,p_3+t_3;q_1+u_1,q_2+u_2,q_3+u_3;r_1+v_1,r_2+v_2,r_3+v_3\right)$.

By applying equation (4), we get

$$V_T\left(\tilde{a}+\tilde{b}\right)=\frac{\left(p_1+t_1\right)+4\left(p_2+t_2\right)+\left(p_3+t_3\right)}{6}$$
$$=\frac{p_1+4p_2+p_3}{6}+\frac{t_1+4t_2+t_3}{6}$$
$$=V_T\left(\tilde{A}\right)+V_T\left(\tilde{B}\right)$$

Hence $V_T\left(\tilde{A}+\tilde{B}\right)=V_T\left(\tilde{A}\right)+V_T\left(\tilde{B}\right)$.

Theorem 3.2: Let $\tilde{A}=\left(p_1,p_2,p_3;q_1,q_2,q_3;r_1,r_2,r_3\right)$ and $\tilde{B}=\left(t_1,t_2,t_3;u_1,u_2,u_3;v_1,v_2,v_3\right)$ be any two TrSVNNs. Then the equation $V_I\left(\tilde{A}+\tilde{B}\right)=V_I\left(\tilde{A}\right)+V_I\left(\tilde{B}\right)$ is valid.

Proof: By applying equation (5), we get

$$V_I\left(\tilde{a}+\tilde{b}\right)=\frac{\left(q_1+u_1\right)+4\left(q_2+u_2\right)+\left(q_3+u_3\right)}{6}$$
$$=\frac{q_1+4q_2+q_3}{6}+\frac{u_1+4u_2+u_3}{6}$$
$$=V_I\left(\tilde{A}\right)+V_I\left(\tilde{B}\right)$$

Hence $V_I\left(\tilde{A}+\tilde{B}\right)=V_I\left(\tilde{A}\right)+V_I\left(\tilde{B}\right)$.

Theorem 3.3: Let $\tilde{A}=\left(p_1,p_2,p_3;q_1,q_2,q_3;r_1,r_2,r_3\right)$ and $\tilde{B}=\left(t_1,t_2,t_3;u_1,u_2,u_3;v_1,v_2,v_3\right)$ be any two TrSVNNs. Then the equation $V_F\left(\tilde{A}+\tilde{B}\right)=V_F\left(\tilde{A}\right)+V_F\left(\tilde{B}\right)$ is valid.

Proof: By applying equation (6), we get

$$V_F\left(\tilde{a}+\tilde{b}\right)=\frac{(r_1+v_1)+4(r_2+u_2)+(r_3+u_3)}{6}$$
$$=\frac{r_1+4r_2+r_3}{6}+\frac{u_1+4u_2+u_3}{6}$$
$$=V_F\left(\tilde{A}\right)+V_F\left(\tilde{B}\right)$$

Hence $V_F\left(\tilde{A}+\tilde{B}\right)=V_F\left(\tilde{A}\right)+V_F\left(\tilde{B}\right)$.

Theorem 3.4: Let $\tilde{A}=(p_1,p_2,p_3;q_1,q_2,q_3;r_1,r_2,r_3)$ and $\tilde{B}=(t_1,t_2,t_3;u_1,u_2,u_3;v_1,v_2,v_3)$ be any two TrSVNNs. Then the equations hold.

(1) $A_T\left(\tilde{A}+\tilde{B}\right)=A_T\left(\tilde{A}\right)+A_T\left(\tilde{B}\right)$

(2) $A_I\left(\tilde{A}+\tilde{B}\right)=A_I\left(\tilde{A}\right)+A_I\left(\tilde{B}\right)$

(3) $A_F\left(\tilde{A}+\tilde{B}\right)=A_F\left(\tilde{A}\right)+A_F\left(\tilde{B}\right)$

Definition 3.6: Let $\tilde{A}=(p_1,p_2,p_3;q_1,q_2,q_3;r_1,r_2,r_3)$ be a TrSVNN. A value index and an ambiguity index for the TrSVNN $\tilde{A}$ are defined as follows:

$$V\left(\tilde{A},(\lambda,\mu,\nu)\right)=\lambda V_T\left(\tilde{A}\right)+\mu V_I\left(\tilde{A}\right)+\nu V_F\left(\tilde{A}\right) \qquad (13)$$

and

$$A\left(\tilde{A},(\lambda,\mu,\nu)\right)=\lambda A_T\left(\tilde{A}\right)+\mu A_I\left(\tilde{A}\right)+\nu A_F\left(\tilde{A}\right) \qquad (14)$$

Here the notation λ,μ and ν that exists in the value and ambiguity indexes represents a weight which indicates the decision maker's preference information with the condition $\lambda+\mu+\nu=1$. . The decision maker may intend to take decision pessimistically in uncertain

environment for $\lambda\in\left[0,\frac{1}{3}\right]$ and $\mu+\nu\in\left[\frac{1}{3},1\right]$. The conditions $\lambda\in\left[\frac{2}{3},1\right]$ and $\mu+\nu\in\left[0,\frac{1}{3}\right]$ shows that the decision maker prefers optimistic feeling in uncertain environment. $\lambda=\mu=\nu=\frac{1}{3}$ shows that the decision maker feels that the degrees of truth, indeterminacy and falsity have same impact. Therefore, the value index and the ambiguity index may reflect the decision maker's subjectivity attitude to the TrSVNNs.

Theorem 3.5: Let $\tilde{A}=(p_1,p_2,p_3;q_1,q_2,q_3;r_1,r_2,r_3)$ and $\tilde{B}=(t_1,t_2,t_3;u_1,u_2,u_3;v_1,v_2,v_3)$ be any two TrSVNNs. Then for $\lambda,\mu,\nu\in[0,1]$ and $k\in\mathbb{R}$, we have

$$V\left(\tilde{A}+\tilde{B},(\lambda,\mu,\nu)\right)=V\left(\tilde{A},(\lambda,\mu,\nu)\right)+V\left(\tilde{B},(\lambda,\mu,\nu)\right) \quad \text{and} \quad V\left(k\tilde{A},(\lambda,\mu,\nu)\right)=kV\left(\tilde{A},(\lambda,\mu,\nu)\right).$$

Proof: For the given TrSVNNs, by the definition of arithmetic operations, we have

$\tilde{A}+\tilde{B}=(p_1+t_1,p_2+t_2,p_3+t_3;q_1+u_1,q_2+u_2,q_3+u_3;r_1+v_1,r_2+v_2,r_3+v_3)$ and

$\lambda\tilde{A}=\left(\lambda p_1,\lambda p_2,\lambda p_3;\lambda q_1,\lambda q_2,\lambda q_3;\lambda r_1,\lambda r_2,\lambda r_3\right).$

Employing these operations, the two statements of the theorem are proved.

$$\begin{aligned}
&V\left(\tilde{A}+\tilde{B},(\lambda,\mu,\nu)\right)=\lambda V_T\left(\tilde{A}+\tilde{B}\right)+\mu V_I\left(\tilde{A}+\tilde{B}\right)+\nu V_F\left(\tilde{A}+\tilde{B}\right)\\
&=\frac{1}{6}\left[\lambda\left\{\left(p_1+t_1\right)+4\left(p_2+t_2\right)+\left(p_3+t_3\right)\right\}\right]+\frac{1}{6}\left[\mu\left\{\left(q_1+u_1\right)+4\left(q_2+u_2\right)+\left(q_3+u_3\right)\right\}\right]+\frac{1}{6}\left[\nu\left\{\left(r_1+v_1\right)+4\left(r_2+v_2\right)+\left(r_3+v_3\right)\right\}\right]\\
&=\frac{1}{6}\left[\lambda\left(p_1+4p_2+p_3+t_1+4t_2+t_3\right)+\mu\left(q_1+4q_2+q_3+u_1+4u_2+u_3\right)+\nu\left(r_1+4r_2+r_3+v_1+4v_2+v_3\right)\right]\\
&=\frac{1}{6}\left[\lambda\left[(p_1+4p_2+p_3)+(t_1+4t_2+t_3)\right]+\mu\left[(q_1+4q_2+q_3)+(u_1+4u_2+u_3)\right]+\nu\left[\left(r_1+4r_2+r_3\right)+\left(v_1+4v_2+v_3\right)\right]\right]\\
&=\left[\lambda\left(\frac{p_1+4p_2+p_3}{6}\right)+\mu\left(\frac{q_1+4q_2+q_3}{6}\right)+\nu\left(\frac{r_1+4r_2+r_3}{6}\right)\right]+\left[\lambda\left(\frac{t_1+4t_2+t_3}{6}\right)+\mu\left(\frac{u_1+4u_2+u_3}{6}\right)+\nu\left(\frac{v_1+4v_2+v_3}{6}\right)\right]\\
&=\left[\lambda V_T\left(\tilde{A}\right)+\mu V_I\left(\tilde{A}\right)+\nu V_F\left(\tilde{A}\right)\right]+\left[\lambda V_T\left(\tilde{B}\right)+\mu V_I\left(\tilde{B}\right)+\nu V_F\left(\tilde{B}\right)\right]\\
&=V\left(\tilde{A},(\lambda,\mu,\nu)\right)+V\left(\tilde{B},(\lambda,\mu,\nu)\right)
\end{aligned}$$

Hence $V\left(\tilde{A}+\tilde{B},(\lambda,\mu,\nu)\right)=V\left(\tilde{A},(\lambda,\mu,\nu)\right)+V\left(\tilde{B},(\lambda,\mu,\nu)\right).$

Let us prove the second part of the theorem as follows:

$$\begin{aligned}
V\left(k\tilde{A},(\lambda,\mu,\nu)\right)&=\lambda V_T\left(k\tilde{A}\right)+\mu V_I\left(k\tilde{A}\right)+\nu V_F\left(k\tilde{A}\right)\\
&=\lambda\left(k\frac{p_1+4p_2+p_3}{6}\right)+\mu\left(k\frac{q_1+4q_2+q_3}{6}\right)+\nu\left(k\frac{r_1+4r_2+r_3}{6}\right)\\
&=\left[\lambda\left(\frac{kp_1+4kp_2+kp_3}{6}\right)+\mu\left(\frac{kq_1+4kq_2+kq_3}{6}\right)+\nu\left(\frac{kr_1+4kr_2+kr_3}{6}\right)\right]\\
&=k\left[\lambda\left(\frac{p_1+4p_2+p_3}{6}\right)+\mu\left(\frac{q_1+4q_2+q_3}{6}\right)+\nu\left(\frac{r_1+4r_2+r_3}{6}\right)\right]\\
&=k\left[\lambda V_T\left(\tilde{A}\right)+\mu V_I\left(\tilde{A}\right)+\nu V_F\left(\tilde{A}\right)\right]\\
&=kV\left(\tilde{A},(\lambda,\mu,\nu)\right)
\end{aligned}$$

Hence $V\left(k\tilde{A},(\lambda,\mu,\nu)\right)=kV\left(\tilde{A},(\lambda,\mu,\nu)\right).$

Theorem 3.6: Let $\tilde{A}=\left(p_1,p_2,p_3;q_1,q_2,q_3;r_1,r_2,r_3\right)$ and $\tilde{B}=\left(t_1,t_2,t_3;u_1,u_2,u_3;v_1,v_2,v_3\right)$ be any two TrSVNNs. Then for $\lambda,\mu,\nu\in[0,1]$ and $k\in\mathbb{R},k>0$ we have

$A\left(\tilde{A}+\tilde{B},(\lambda,\mu,\nu)\right)=A\left(\tilde{A},(\lambda,\mu,\nu)\right)+A\left(\tilde{B},(\lambda,\mu,\nu)\right)$ and $A\left(k\tilde{A},(\lambda,\mu,\nu)\right)=kA\left(\tilde{A},(\lambda,\mu,\nu)\right).$

Proof: For the given TrSVNNs, by the definition of arithmetic operations, we have

$\tilde{A}+\tilde{B}=\left(p_1+t_1,p_2+t_2,p_3+t_3;q_1+u_1,q_2+u_2,q_3+u_3;r_1+v_1,r_2+v_2,r_3+v_3\right)$ and

$\lambda\tilde{A}=\left(\lambda p_1,\lambda p_2,\lambda p_3;\lambda q_1,\lambda q_2,\lambda q_3;\lambda r_1,\lambda r_2,\lambda r_3\right).$

$$A\left(\tilde{A}+\tilde{B},(\lambda,\mu,\nu)\right)=\lambda A_T\left(\tilde{A}+\tilde{B}\right)+\mu A_I\left(\tilde{A}+\tilde{B}\right)+\nu A_F\left(\tilde{A}+\tilde{B}\right)$$
$$=\frac{1}{6}\left[\lambda\left\{(p_3+t_3)-(p_1+t_1)\right\}\right]+\frac{1}{6}\left[\mu\left\{(q_3+u_3)-(q_1+u_1)\right\}\right]+\frac{1}{6}\left[\nu\left\{(r_3+v_3)-(r_1+v_1)\right\}\right]$$
$$=\frac{1}{6}\left[\lambda(p_3+t_3-p_1-t_1)+\mu(q_3+u_3-q_1-u_1)+\nu(r_3+v_3-r_1-v_1)\right]$$
$$=\left[\lambda\left(\frac{p_3-p_1}{6}\right)+\mu\left(\frac{q_3-q_1}{6}\right)+\nu\left(\frac{r_3-r_1}{6}\right)\right]+\left[\lambda\left(\frac{t_3-t_1}{6}\right)+\mu\left(\frac{u_3-u_1}{6}\right)+\nu\left(\frac{v_3-v_1}{6}\right)\right]$$
$$=\left[\lambda A_T\left(\tilde{A}\right)+\mu A_I\left(\tilde{A}\right)+\nu A_F\left(\tilde{A}\right)\right]+\left[\lambda A_T\left(\tilde{B}\right)+\mu A_I\left(\tilde{B}\right)+\nu A_F\left(\tilde{B}\right)\right]$$
$$=A\left(\tilde{A},(\lambda,\mu,\nu)\right)+A\left(\tilde{B},(\lambda,\mu,\nu)\right)$$

Hence $A\left(\tilde{A}+\tilde{B},(\lambda,\mu,\nu)\right)=A\left(\tilde{A},(\lambda,\mu,\nu)\right)+A\left(\tilde{B},(\lambda,\mu,\nu)\right)$.

Let us prove the second part of the theorem as follows:

$$A\left(k\tilde{A},(\lambda,\mu,\nu)\right)=\lambda A_T\left(k\tilde{A}\right)+\mu A_I\left(k\tilde{A}\right)+\nu A_F\left(k\tilde{A}\right)$$
$$=\left[\lambda\left(\frac{-kp_1+kp_3}{6}\right)+\mu\left(\frac{-kq_1+kq_3}{6}\right)+\nu\left(\frac{-kr_1+kr_3}{6}\right)\right]$$
$$=k\left[\lambda\left(\frac{-p_1+p_3}{6}\right)+\mu\left(\frac{-q_1+q_3}{6}\right)+\nu\left(\frac{-r_1+r_3}{6}\right)\right]$$
$$=k\left[\lambda A_T\left(\tilde{A}\right)+\mu A_I\left(\tilde{A}\right)+\nu A_F\left(\tilde{A}\right)\right]$$
$$=kA\left(\tilde{A},(\lambda,\mu,\nu)\right)$$

Hence $A\left(k\tilde{A},(\lambda,\mu,\nu)\right)=kA\left(\tilde{A},(\lambda,\mu,\nu)\right)$.

Theorem 3.7: Let $\tilde{A}=(p_1,p_2,p_3;q_1,q_2,q_3;r_1,r_2,r_3)$ and $\tilde{B}=(t_1,t_2,t_3;u_1,u_2,u_3;v_1,v_2,v_3)$ be any two TrSVNNs. If $p_1>v_3$, then $\tilde{A}>\tilde{B}$.

Proof: From the results (4), (5) and (6), we obtain

$$V_T\left(\tilde{A}\right)=\int_0^1\left[L_{\tilde{A}}(\alpha)+R_{\tilde{A}}(a)\right]f(\alpha)\,d\alpha$$
$$=\frac{p_1+4p_2+p_3}{6}>p_1$$

$$V_I\left(\tilde{A}\right)=\int_0^1\left[L_{\tilde{A}}(\beta)+R_{\tilde{A}}(\beta)\right]g(\beta)\,d\beta$$
$$=\frac{q_1+4q_2+q_3}{6}>p_1$$

$$V_F\left(\tilde{A}\right)=\int_0^1\left[L_{\tilde{A}}(\gamma)+R_{\tilde{A}}(\gamma)\right]h(\gamma)\,d\gamma$$
$$=\frac{r_1+4r_2+r_3}{6}>p_1$$

$$V_T\left(\tilde{B}\right)=\int_0^1\left[L_{\tilde{B}}(\alpha)+R_{\tilde{B}}(a)\right]f(\alpha)d\alpha$$
$$=\frac{t_1+4t_2+t_3}{6}<v_3$$

$$V_I\left(\tilde{B}\right)=\int_0^1\left[L_{\tilde{B}}(\beta)+R_{\tilde{B}}(\beta)\right]g(\beta)d\beta$$
$$=\frac{u_1+4u_2+u_3}{6}<v_3$$

$$V_F\left(\tilde{B}\right)=\int_0^1\left[L_{\tilde{B}}(\gamma)+R_{\tilde{B}}(\gamma)\right]h(\gamma)d\gamma$$
$$=\frac{v_1+4v_2+v_3}{6}<v_3$$

Combining with the condition $p_1>v_3$, we get $V_T\left(\tilde{A}\right)>V_T\left(\tilde{B}\right), V_I\left(\tilde{A}\right)>V_I\left(\tilde{B}\right)$ and $V_F\left(\tilde{A}\right)>V_F\left(\tilde{B}\right)$.

By applying the definition (3.6) and for any values of $\lambda,\mu,\nu\in[0,1]$, we have

$$V\left(\tilde{A},(\lambda,\mu,\nu)\right)=\lambda V_T\left(\tilde{A}\right)+\mu V_I\left(\tilde{A}\right)+\nu V_F\left(\tilde{A}\right)$$
$$>\lambda V_T\left(\tilde{B}\right)+\mu V_I\left(\tilde{B}\right)+\nu V_F\left(\tilde{B}\right)=V\left(\tilde{B},(\lambda,\mu,\nu)\right)$$

Thus $V\left(\tilde{A},(\lambda,\mu,\nu)\right)>V\left(\tilde{B},(\lambda,\mu,\nu)\right)$

Hence $\tilde{A}>\tilde{B}$.

3.3 Ranking Procedure:

In this section, ratio of value and ambiguity indices is defined and a ranking based on this ratio was provided to rank TrSVNNs.

Definition 3.7: A ratio of the value index to the ambiguity index for a TrSVNN $\tilde{A}=(p_1,p_2,p_3;q_1,q_2,q_3;r_1,r_2,r_3)$ is defined as follows:

$$R\left(\tilde{A},(\lambda,\mu,\nu)\right) = \frac{V\left(\tilde{A},(\lambda,\mu,\nu)\right)}{1+A\left(\tilde{A},(\lambda,\mu,\nu)\right)} \tag{15}$$

The above equation, clearly shows that $R\left(\tilde{A},(\lambda,\mu,\nu)\right)$ is not a linear function of a TIFN $\tilde{A}$ although both $V\left(\tilde{A},(\lambda,\mu,\nu)\right), A\left(\tilde{A},(\lambda,\mu,\nu)\right)$ are linear on $\tilde{A}$.

Ranking Algorithm:

Let $\tilde{A}_i=(p_{i1},p_{i2},p_{i3};q_{i1},q_{i2},q_{i3};r_{i1},r_{i2},r_{i3}),\ i=1,2,3,.....n$ be a list of ‘n’ TrSVNNs. A ranking procedure based on the ratio ranking defined in equation (15) is explained as follows:

Step 1: In the first step, find the value and ambiguity of truth, indeterminacy and falsity functions $V_T\left(\tilde{A}_i\right), V_I\left(\tilde{A}_i\right), V_F\left(\tilde{A}_i\right), A_T\left(\tilde{A}_i\right), A_I\left(\tilde{A}_i\right)$ and $A_F\left(\tilde{A}_i\right)$, for $i = 1, 2, 3, ..n$ using the equations (4-6) and (10-12).

Step 2: Compute the value and ambiguity indices respectively $V\left(\tilde{A}_i, (\lambda, \mu, \nu)\right)$ and $A\left(\tilde{A}_i, (\lambda, \mu, \nu)\right)$ of the given 'n' TrSVNNs using the relations produced by the equations (13) and (14).

Step 3: Compute the ratio namely $R\left(\tilde{A}_i, (\lambda, \mu, \nu)\right)$ of individual TrSVNNs by applying the relation given by (15), for the same values of λ, μ and ν.

Step 4: Rank the given set of 'n' TrSVNNs $\tilde{A}_i$, for $i = 1, 2, 3, ...n$ according to non-increasing order of the ratios $R\left(\tilde{A}_i, (\lambda, \mu, \nu)\right) / i = 1, 2, \ldots, n$. The maximum TIFN is the one with the largest ratio, i.e., $max\left\{R\left(\tilde{A}_i, (\lambda, \mu, \nu)\right) / i = 1, 2, \ldots, n\right\}$.

4. Numerical Examples

In this section some numerical examples were presented to illustrate the above proposed ranking algorithm.

Example 4.1: Consider a TrSVNN $\tilde{A} = (10, 15, 20; 14, 16, 22; 12)$, Let us find the value and ambiguity of truth, indeterminacy and falsity membership functions.

From equations (4-6), we get $V_T\left(\tilde{A}\right) = 15$, $V_I\left(\tilde{A}\right) = 16.67$, $V_F\left(\tilde{A}\right) = 15.17$

Also using equations (10-12), we get $A_T\left(\tilde{A}\right) = 1.67$, $A_I\left(\tilde{A}\right) = 1.33$, $A_F\left(\tilde{A}\right) = 1.17$.

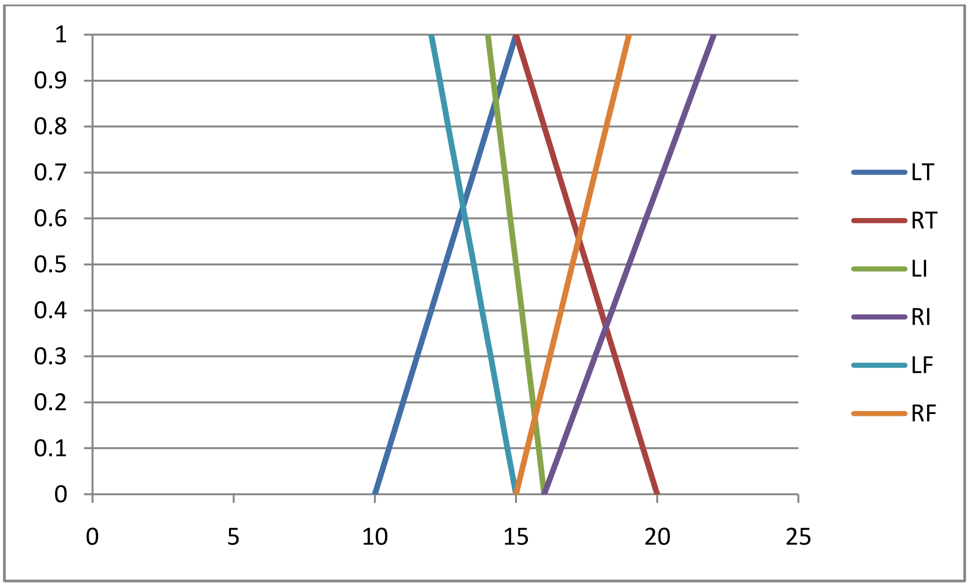

Figure 4.1 Representation of TrSVNN $\tilde{A}$

The value index and ambiguity index of each alternative have been examined for different values for λ, μ, $\nu \in [0, 1]$. The results have been shown in the Table – 4.1.

S.NO	Values of λ,μ,ν	Value Index	Ambiguity Index	Ratio
1	λ=1/6,μ=3/6,ν=2/6	15.89	1.33	6.81
2	λ=0.15,μ=0.35,ν=0.50	15.67	1.30	6.81
3	λ=1/3,μ=1/3,ν=1/3	10.01	1.38	4.20
4	λ=0.70,μ=0.10,ν=0.20	15.20	1.54	5.99
5	λ=0.50,μ=0.30,ν=0.20	15.54	1.47	6.29

Table 4.1 Value, Ambiguity and Ratio indices of $\tilde{A}$

Example 4.2: In this example 2 TrSVNNs are to be ordered by employing the ratio ranking approach. Consider two TrSVNNs $\tilde{A} = (10,15,20;14,16,22;12,15,19)$ and $\tilde{B} = (5,9,14;8,11,18;7,13,16)$. The value, ambiguity and ration indices of TrSVNN $\tilde{B}$ are summarized in the following table 4.2.

S.NO	Values of λ,μ,ν	Value Index	Ambiguity Index	Ratio
1	λ=1/6,μ=3/6,ν=2/6	11.53	1.58	4.46
2	λ=0.15,μ=0.35,ν=0.50	11.71	1.56	4.58
3	λ=1/3,μ=1/3,ν=1/3	11.00	1.54	4.33
4	λ=0.70,μ=0.10,ν=0.20	10.08	1.52	4.01
5	λ=0.50,μ=0.30,ν=0.20	10.58	1.55	4.15

Table 4.1 Value, Ambiguity and Ratio indices of $\tilde{B}$

By comparing the last columns of tables 4.1 and 4.2, we see that $R\left(\tilde{A},(\lambda,\mu,\nu)\right) > R\left(\tilde{B},(\lambda,\mu,\nu)\right)$, irrespective of the values of λ, μ and ν. Hence $\tilde{A} \succ \tilde{B}$.

5. Conclusion

In recent years many ranking methods were developed for ordering of neutrosophic numbers which plays a vital role in linear, non-linear programming problems, multiple attribute decision making problems and engineering applications. In this article ranking of triangular single valued neutrosophic numbers based on ratio of value and ambiguity indices was presented. So far in the literature, there are no ranking methods developed so far to rank triangular single value neutrosophic numbers of type 1. A comparative study of the proposed algorithm cannot be done in the present form. As an extension in future work, this ordering will be applied to solve uncertainty based problems.

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