OUTER SUM LABELLING OF WHEEL

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Abstract. Let An Outer sum labeling is a labeling of a graph G is an injective function $f : V(G) \longrightarrow Z^+$ With the property that each vertex $v \in V(G)$ there exist a vertex $w \in V(G)$ Such that $f(w) = \sum_{u \in N(V)} f(u), N(v) = \{x : vx \in V(G)\}$ A graph G which admits an outer sum labeling is called an outer sum graph. If G is not an outer sum graph then the minimum of isolated vertices required to make G a outer sum graph, is called outer sum number of G and is denoted by $on(G)$. In this paper we determine outer sum number of wheels.

Key words: Sum labeling, Sum graphs, Sum number, Outer sum labeling , outer sum graphs ,outer sum number
AMS classification: 05C78,05C12,05C15.

1. INTRODUCTION

Lots of research work is been carried out in the labeling of graphs in past few work since the first initiated by A.Rosa [1].A graph labeling is the assignment of labels,traditionally represented by integers,to the edges or vertices, or both,of a graph.[5] Formally given a graph $G = (V, E)$, a vertex labeling is a function of V to a set of labels. A graph with such a function defined is called a vertex-labeled graph. Likewise, an edge labeling is a function of E to a set of labels. In this case, the graph is called an edge-labeled graph. Due to development of information technology and solving typical algorithms of coloring problems graph labeling is extremely useful. For a large network of transmitters spread out in a planar region, the channel assignment problem is to assign a numerical channel representing frequency, to each transmitter. The channels assigned to nearby transmitters satisfy some separation constraints so as to avoid interference. The goal is to minimize the portion of the frequency spectrum that must be allocated to the problem, so that it is desire to minimize the span(the largest frequency) of the feasible assignment.
We begin with simple, finite, connected, undirected and non-trivial graph G = (V,E), where V is called the set of vertices and E is called the set of edges.For various graph theoretic notations and terminology we follow Gross and Yellen [9]and for number theory we follow Burton[4]. We will give brief summery of definitions which are useful for the present investigations.

Definition 1.1: If the vertices of the graph are assigned values, subject to certain conditions is known as graph labeling. For detail survey on graph labeling one can refer Gallian [10]. Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in last four decades.Most interesting labeling problems have three important ingredients.
(i) a set of numbers from which vertex labels are chosen.
(ii) a rule that assigns a value to each edge.
(iii)a condition that these values must satisfy.

Definition 1.2: The helm graph H_n is the graph obtained from a wheel $W_n = C_n + K_1$ by attaching a pendant edge at each vertex of C_n.

Definition 1.3:A chord of a cycle C_n is an edge joining two non-adjacent vertices of cycle C_n.

2. SUM GRAPHS

A sum labeling λ of a graph is a mapping of the vertices of G into distinct positive integers such that for $u, v \in V(G), uv \in E(G)$ if and only if $\lambda(u) + \lambda(v) = \lambda(w)$ for some vertex w of G. A graph which admits a sum labeling is called a sum graph.Sum graph were originally proposed by F. Harary [6] and later he extended to include all integers in [7].

Sum graphs cannot be connected graphs since an edge from the vertex with the largest label would necessitate a vertex with a larger label. Graphs which are not sum graphs can be made to support a sum labeling by considering the graph in conjunction with a number of isolated vertices which can bear the labels required by the graph. The fewest number of the additional isolates required by the graph support to sum labeling is called the **sum number** of the graph, it is denoted by $\sigma(G)$.

The length of a shortest path between two vertices u and v in a graph G is called the distance between u and v and is denoted by $d(u, v)$.

3. OUTER SUM GRAPHS

An outer sum labeling f of a non-trivial graph is a mapping of the vertices of G into distinct positive integers such that for each vertex $v \in V(G)$, there exists a vertex $w \in V(G)$ with $f(w) = \sum_{u \in N(v)} f(u)$, where $N(v) = x : vx \in E(G)$. A graph G which admits an outer sum labeling is called an outer sum graph.If G is not an outer sum graph, then by adding certain number of isolated vertices to G,we can make the resultant graph an outer sum graph.The minimum of such isolated vertices required for a graph G, to make the resultant graph an outer sum graph, is called the **outer sum number** of G and is denoted by $on(G)$. Outer sum graph were originally proposed by B. Sooryanarayana, Manjula K and Vishu Kumar M [3].

4. Known results

Remark. *A graph is an outer sum graph iff its outer sum number is zero.*

Remark. *For any graph G on n vertices with at most (n - 2) pendant vertices, we see that there are at least two non terminal vertices u and v that are adjacent in G.*

Theorem 1. *A connected graph G is an outer sum graph if and only if* $G \equiv K_{1,n}$.

Theorem 2. *For any integer* $n \geq 3$, $on(C_n) = \begin{cases} 1 & if \quad n = 4 \\ 2 & otherwise \end{cases}$

Theorem 3. *Outer sum number of every unicyclic graph containing at least one pendant vertex is 1.*

Theorem 4. *For any tree T on n vertices,* $on(T) = \begin{cases} 0 & ifTisastar \\ 1 & otherwise \end{cases}$

Corollary 4.1. *For any connected graph* $G(V, E)$, $On(G) \leq 2(|E| - |V|) + 3$

Theorem 5. *For any positive integer n, the outer sum number of a complete graph* K_n *is* $on(K_n) = \begin{cases} 0 & if \quad n \leq 2 \\ n - 1 & otherwise \end{cases}$

Theorem 6. *For any positive integers $m_1 \leq m_2, ... \leq m_k$ the outer sum number of a complete k-partite graph $K_{m_1,m_2,m_3,....m_k}$ is given by*

$$on(m_1, m_2, m_3, ...m_k) = \begin{cases} 0, & if \quad m_1 = 1 \quad and \quad k = 2 \\ 1, & if \quad m_i \neq 1 \quad forall \quad i \geq 2 \\ i, & if \quad m_i = 1 \quad mi + 1 > 1, \quad for \quad any \quad i \quad 1 < i < k \end{cases}$$

Theorem 7. *For any integer $n \geq 1$, $on(K_{1,n} + k_1) = 2$.*

Theorem 8. *For any positive integer n, $on(P_n + K_1) = \begin{cases} 0, & if & n = 1 \\ 1, & if & n = 3 \\ 2, & otherwise & \end{cases}$*

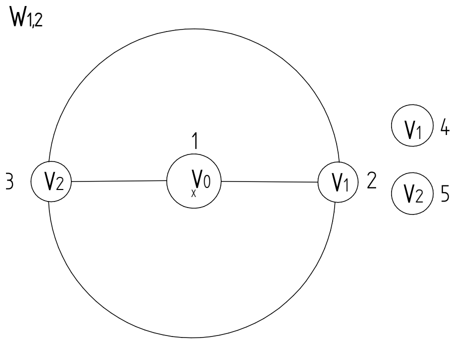
Theorem 9. *For a given integer $n \geq 2$, $on(P_n^{n-2}) = \begin{cases} 1, & if \quad n = 2 \\ 2, & if \quad n = 3 \\ 1, & if \quad n = 4 \\ n-2, & if \quad n \geq 5 \end{cases}$*

5. OUTER SUM GRAPHS OF A WHEELS

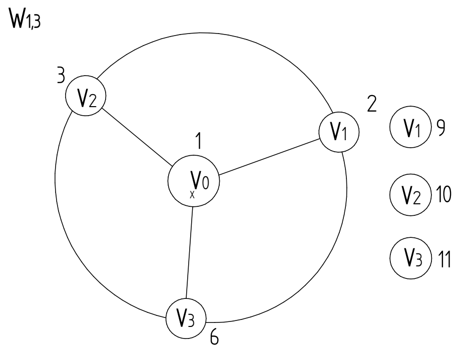
In this section we define an outer sum labeling and compute the outer sum number of wheels. An outer sum labeling of a graph G is a labeling on G with, for each vertex $v \in V(G)$ there exists a vertex $w \in V(G)$ such that $f(w) = \sum_{u \in N(v)} f(u)$, where $N(v) = x : vx \in E(G)$. A graph G which contains an outer sum labeling is called an outer sum graph. If G is not an outer sum graph, then by adding a certain number of isolated vertices in G we can make the resultant graph an outer sum graph. A minimum number of isolated vertices required to make an outer sum graph, is called the outer sum number and is denoted by $on(G)$ .That is the outer sum number of G is the minimum nonnegative integer n such that $G \cup \overline{K_n}$ is an outer sum graph."

Theorem 10. *For any positive integer $n \geq 12$, $on(W_{1,n}) = 3$.*

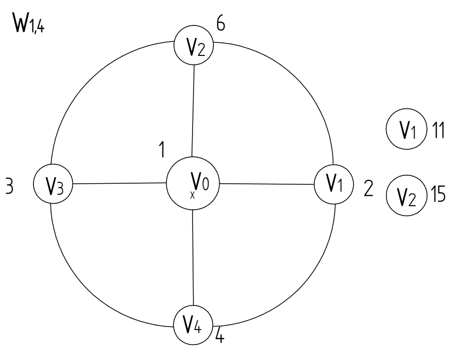
Proof. The case n=2,3,4,5,6,7,8,9,10,11 follows by figures □



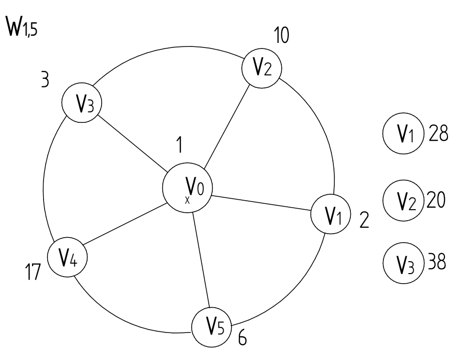
$(a) \quad on(W_{1,2}) = 2$



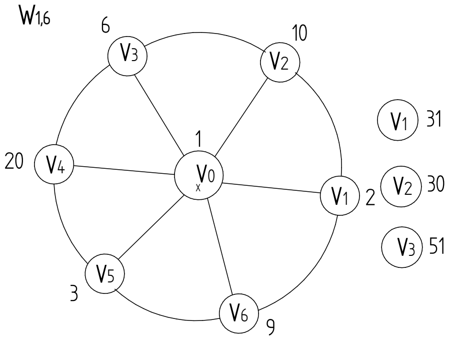
$(b) \quad on(W_{1,3}) = 3$



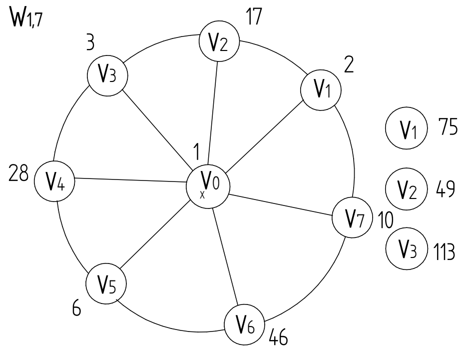
$(c) \quad on(W_{1,4}) = 2$



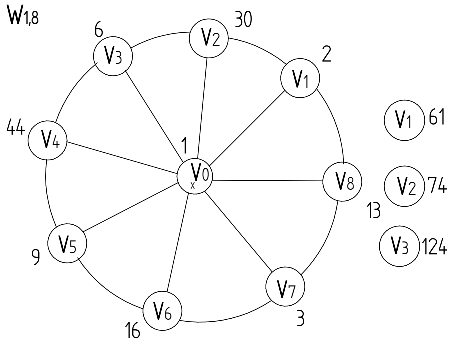
$(d) \quad on(W_{1,5}) = 3$



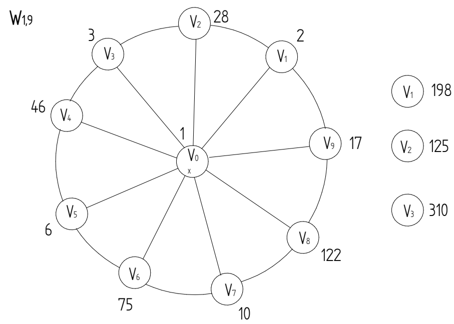
$(e) \quad on(W_{1,6}) = 3$



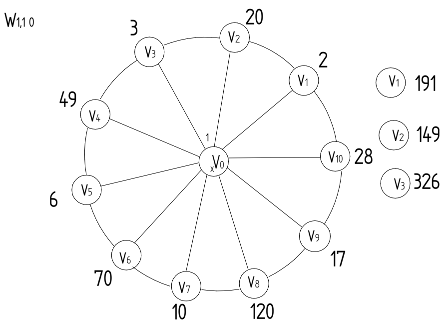
$(f) \quad on(W_{1,7}) = 3$



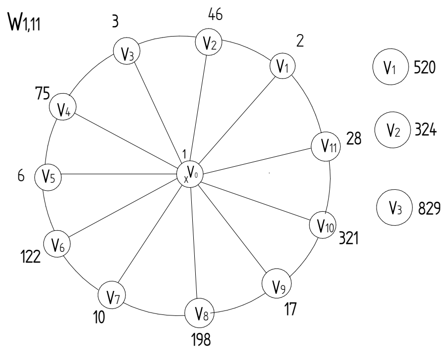
$(g) \quad on(W_{1,8}) = 3$



$(h) \quad on(W_{1,9}) = 3$



$(i) \quad on(W_{1,10}) = 3$



$(j) \quad on(W_{1,11}) = 3$

Figure 1. Outersum number of Wheels with $n < 12$ vertices

By theorem 8[3], $on(P_n + K_1) \geq 2$ for $n \geq 4$, hence $on(W_{1,n}) \geq 2$. Also $\sum f(v_0)$ in $w_{1,n}$ cannot be assigned to any v_i, $1 \leq i \leq n$. Hence one more isolated vertex required to assign $\sum f(v_0)$.
Hence $on(W_{1,n}) \geq 3$.

To prove the reverse inequality we define a labeling f as follows:

Case 1 : When $n = 12, 20, 28, 36, 44, \ldots\ldots\ldots$

- Define $f(v_2) = 2, f(v_4) = 5, f(v_{n-3}) = 3, f(v_{n-5}) = 4, f(x) = 1$
- For $i = 3, 4, 5 \ldots\ldots n/4$
  Define $f(v_{2i}) = f(v_{2i-4}) + f(v_{2i-2}) + 1$
- Define $f(v_{n-2/2}) = f(v_{n/2}) + 1$
- Define $f(v_{n-6/2}) = f(v_{n-4/2}) + f(v_{n/2}) + 1$
- For $i = 2, 3, 4 \ldots\ldots\ldots (n-4)/4$
  Define $f(V_{n/2-(2i+1)}) = f(v_{n/2-(2i+1)+2}) + f(v_{n/2-(2i+1)+4}) + 1$
- For $i = 1, 2, 3, \ldots\ldots\ldots n - 12/4$
  Define $f(V_{n/2+(2i-1)}) = f(v_{n/2-2i}) + (-1)^i$
- For $i = 1, 2, 3, \ldots\ldots\ldots n - 4/4$
  Define $f(V_{n/2+2i}) = f(v_{n/2-(2i+1)}) - (-1)^i$
- Define $f(v_{n-1}) = f(v_1) + f(v_3)$
  $f(v_n) = f(v_{n-1}) + 1$

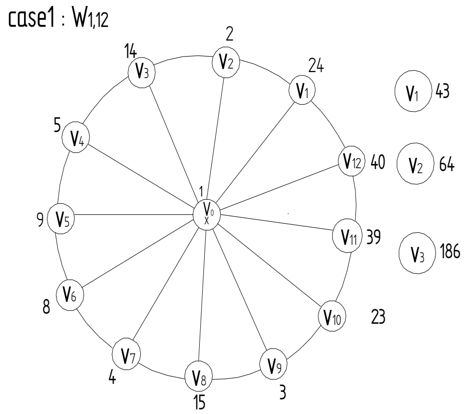


FIGURE 2. $on(W_{1,12}) = 3$

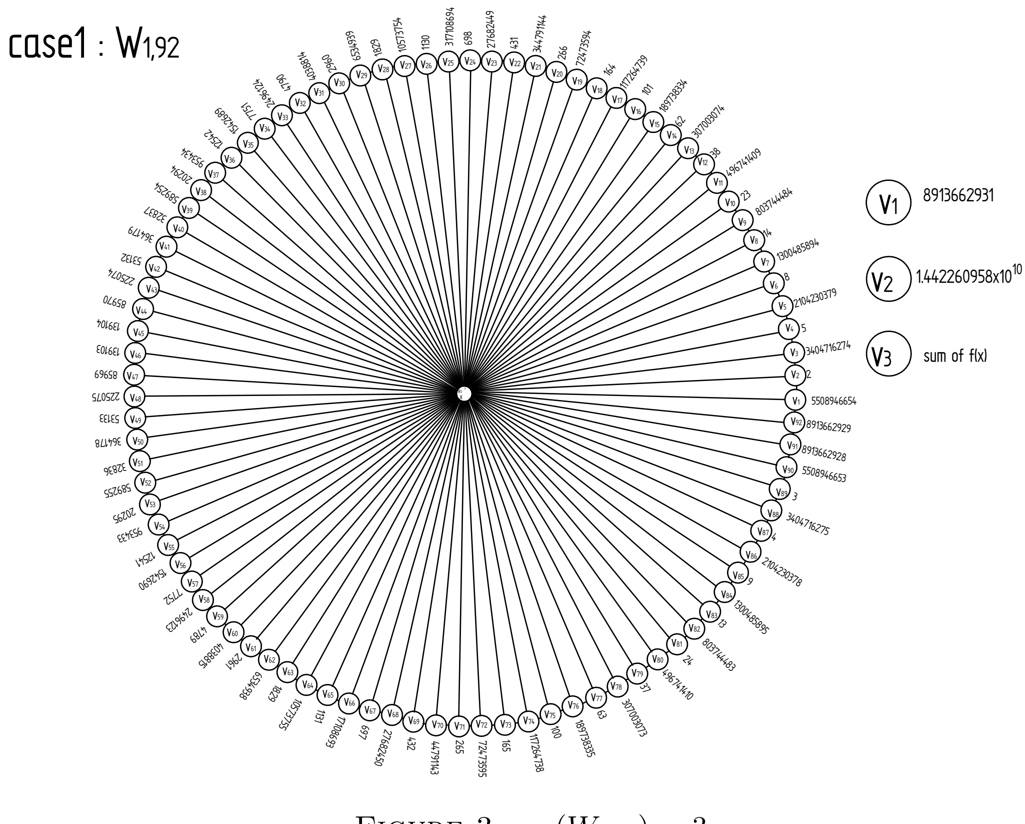


FIGURE 3. $on(W_{1,92}) = 3$

Case 2 : When $n = 13, 21, 29, 37, 45, \ldots\ldots\ldots$

- Define$f(v_2) = 2, f(v_4) = 5, f(v_{n-3}) = 3, f(v_{n-5}) = 4, f(x) = 1$
- For $i = 3, 4, 5......(n-1)/4$
  Define $f(v_{2i}) = f(v_{2i-4}) + f(v_{2i-2}) + 1$
- Define $f(v_{(n-3)/2}) = f(v_{(n-5)/2}) + f(v_{(n-1)/2}) + 1$
- Define $f(v_{(n+1)/2}) = f(v_{(n-3)/2}) - 1$
- For $i = 1, 2, 3, 4.........(n-5)/4$
  Define $f(V_{(n-1)/2-(2i+1)}) = f(v_{(n-1)/2-(2i-1)}) + f(v_{(n-1)/2-(2i-3)}) + 1$
- For $i = 1, 2, 3, 4.........(n-13)/4$
  Define $f(V_{(n-1)/2+2i}) = f(v_{(n-1)/2-2i}) + (-1)^i$
- For $i = 1, 2, 3, 4.........(n-5)/4$
  Define $f(V_{(n-1)/2-(2i+1)}) = f(v_{(n-1)/2-(2i+1)}) - (-1)^i$
- Define $f(v_{n-1}) = f(v_1) + f(v_3)$
  $f(v_n) = f(v_{n-1}) + 1$

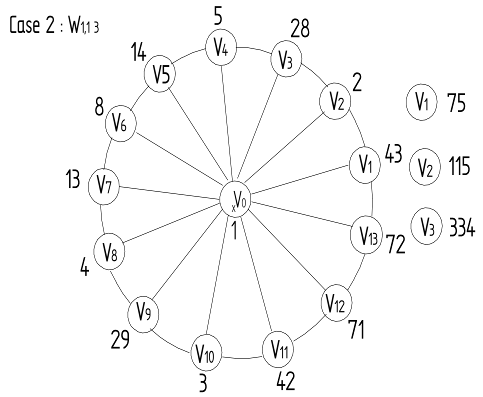


FIGURE 4. $on(W_{1,13}) = 3$

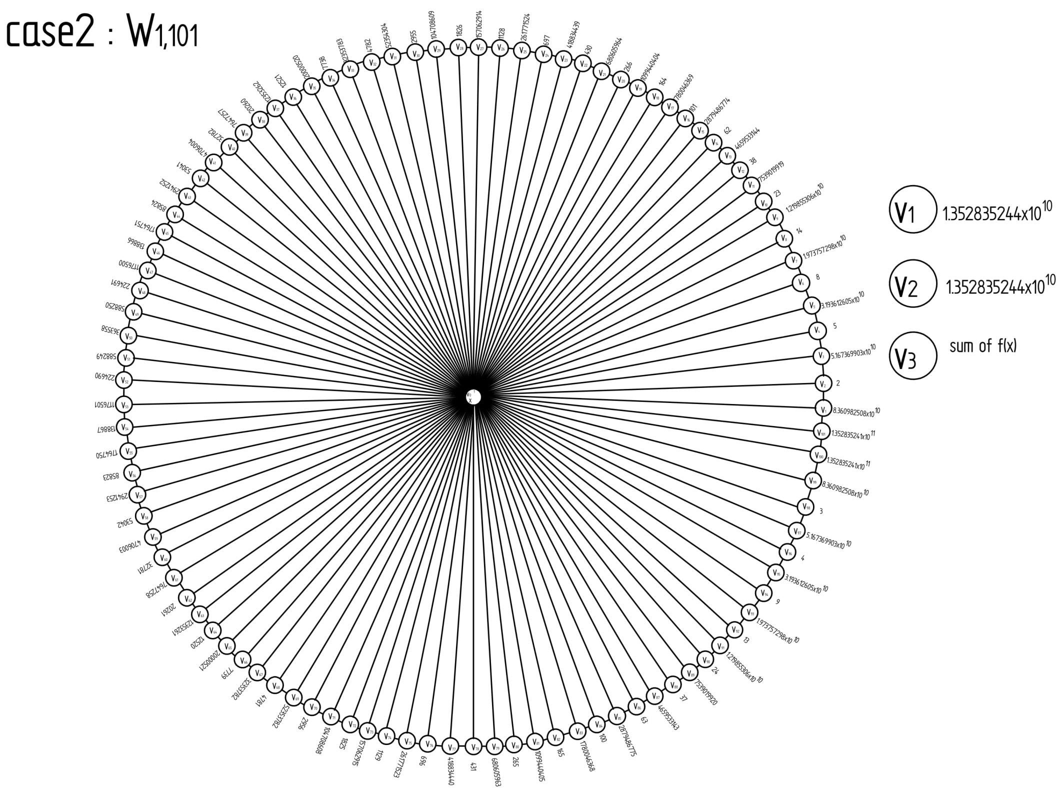


FIGURE 5. $on(W_{1,101}) = 3$

Case 3 : When $n = 14, 22, 30, 38, 46, ...$

- Define $f(v_2) = 2, f(v_4) = 5, f(v_{n-3}) = 3, f(v_{n-5}) = 4, f(x) = 1$
- For $i = 3, 4, 5......(n-6)/4$
  Define $f(v_{2i}) = f(v_{2i-4}) + f(v_{2i-2}) + 1$
- Define $f(v_{(n-2)/2}) = f(v_{(n-6)/2}) + f(v_{(n-10)/2}) + 2$
- Define $f(v_{n/2}) = f(v_{(n-6)/2}) + f(v_{(n-10)/2}) + 1$
- Define $f(v_{(n-4)/2}) = f(v_{(n-6)/2}) + f(v_{(n-2)/2}) + 1$
- Define $f(v_{(n-8)/2}) = f(v_{(n-4)/2}) + f(v_{n/2})$
- For $i = 2, 3, 4......(n-6)/4$
  Define $f(V_{(n-2)/2-(2i+1)}) = f(v_{(n-2)/2-(2i-1)}) + f(v_{(n-2)/2-(2i-3)}) + 1$
- Define $f(v_{(n+2)/2}) = f(v_{(n-4)/2}) - 2$
- For $i = 1, 2, 3, 4.........(n-6)/4$
  Define $f(V_{n/2+(2i+1)}) = f(v_{n/2-(2i+2)}) - (-1)^i$
- For $i = 1, 2, 3, ...(n-14)/4$
  Define $f(V_{n/2+2i}) = f(v_{n/2-(2i+1)}) + (-1)^i$
- Define $f(v_{n-1}) = f(v_1) + f(v_3)$
  $f(v_n) = f(v_{n-1}) + 1$

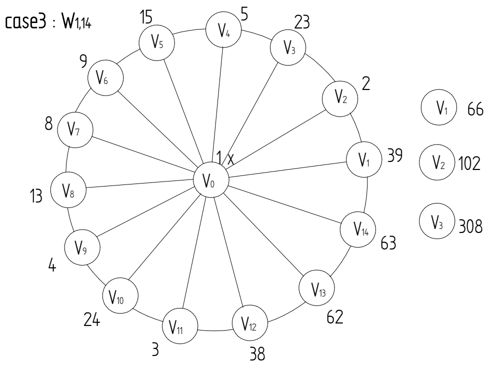


FIGURE 6. $on(W_{1,14}) = 3$

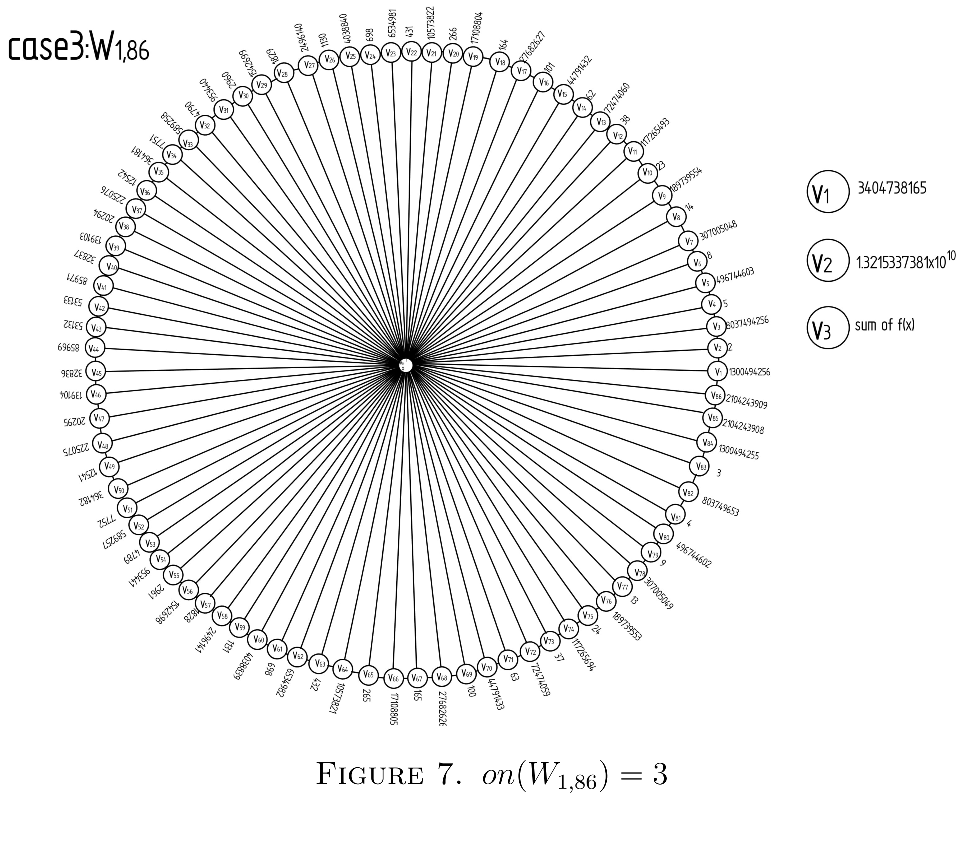


FIGURE 7. $on(W_{1,86}) = 3$

Case 4 : When $n = 15, 23, 31, 39, 47, ...$

- Define $f(v_2) = 2, f(v_4) = 5, f(v_{n-3}) = 3, f(v_{n-5}) = 4, f(x) = 1$
- For $i = 3, 4, 5......(n-3)/4$
  Define $f(v_{2i}) = f(v_{2i-4}) + f(v_{2i-2}) + 1$
- Define $f(v_{(n-1)/2}) = f(v_{(n-3)/2}) + f(v_{(n-7)/2}) + 1$
- Define $f(v_{(n+1)/2}) = f(v_{(n-3)/2}) + 1$
- Define $f(v_{(n-5)/2}) = f(v_{(n-3)/2}) + f(v_{(n+1)/2}) + 2$
- For $i = 1, 2, 3, 4......(n-7)/4$
  Define $f(V_{(n-3)/2-(2i+1)}) = f(v_{(n-3)/2-(2i-1)}) + f(v_{(n-3)/2-(2i-3)}) + [1 + (-1)^i]/2$
- For $i = 1, 2, 3, ...(n-3)/4$
  Define $f(V_{(n-1)/2+2i}) = f(v_{(n-1)/2-2i}) + (-1)^i$
- For $i = 1, 2, 3, 4......(n-7)/8$
  Define $f(V_{(n-1)/2+(2i+1)}) = f(v_{(n-1)/2-(2i+1)}) + (-1)^i$
- Define $f(v_{n-1}) = f(v_1) + f(v_3)$
  $f(v_n) = f(v_{n-1}) + 1$

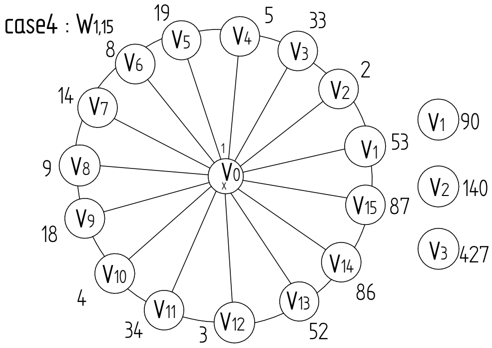


FIGURE 8. $on(W_{1,15}) = 3$

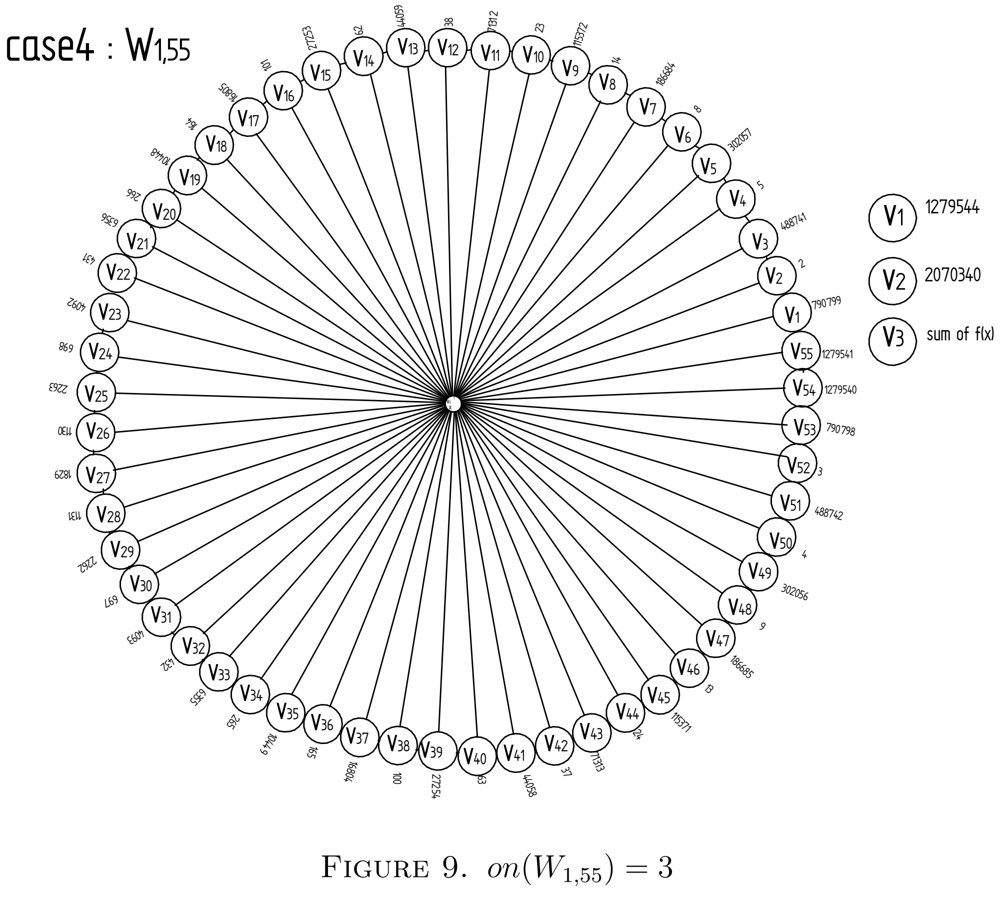


FIGURE 9. $on(W_{1,55}) = 3$

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Case 5 : When $n = 16, 24, 32, 40, 48, ...$

- Define $f(v_2) = 2, f(v_4) = 5, f(v_{n-3}) = 3, f(v_{n-5}) = 4, f(x) = 1$
- For $i = 3, 4, 5......(n-4)/4$
  Define $f(V_{2i}) = f(v_{2i-4}) + f(v_{2i-2}) + 1 + [1 + (-1)^i]/2$
- Define $f(v_{(n-2)/2}) = f(v_{(n-8)/2}) + f(v_{(n-4)/2}) + 1$
- Define $f(v_{n/2}) = f(v_{(n-2)/2}) + 1$
- For $i = 1, 2, 3, 4, 5......(n-8)/4$
  Define $f(V_{n/2+2i}) = f(v_{n/2+(2i-4)}) + f(v_{n/2+(2i-2)}) + 1 + [1 + (-1)^i]/2$
- Define $f(v_{n-2}) = f(v_{n-4}) + f(v_{n-6})$
- For $i = 1, 2, 3, 4.........(n-12)/4$
  Define $f(V_{n/2+(2i-1)}) = f(v_{n/2-2i}) - (-1)^i$
- For $i = 1, 2, 3, 4......(n-4)/4$
  Define $f(V_{n/2-(2i+1)}) = f(v_{n/2+2i}) - (-1)^i$
- Define $f(v_{n-1}) = f(v_1) + f(v_3)$
  $f(v_n) = f(v_{n-1}) + 1$

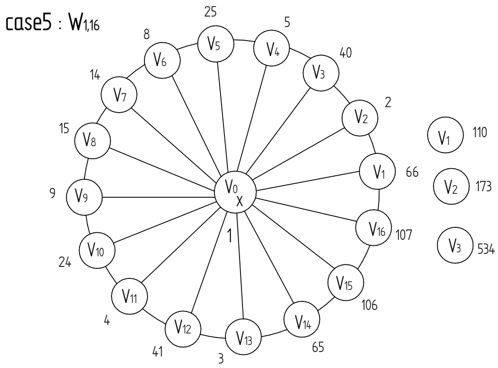

FIGURE 10. $on(W_{1,16}) = 3$

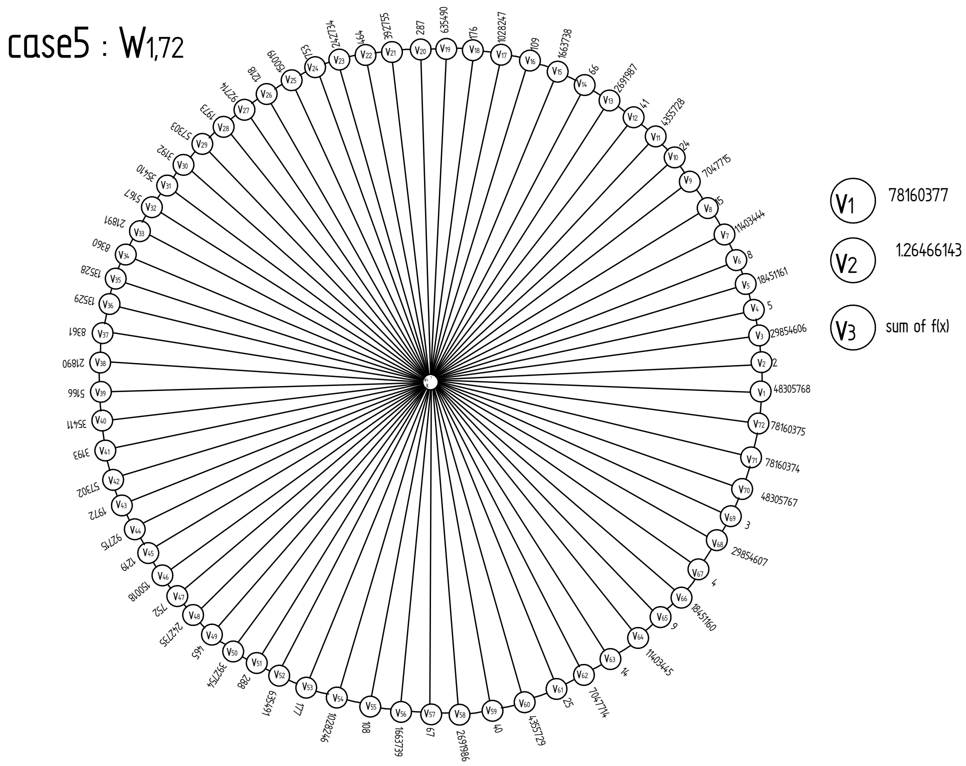

FIGURE 11. $on(W_{1,72}) = 3$

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Case 6 : When $n = 17, 25, 33, 41, 49, ...$

- Define $f(v_2) = 2, f(v_4) = 5, f(v_{n-3}) = 3, f(v_{n-5}) = 4, f(x) = 1$
- For $i = 3, 4, 5......(n-1)/4$
  Define $f(V_{2i}) = f(v_{2i-4}) + f(v_{2i-2}) + 1$
- Define $f(v_{(n-3)/2}) = f(v_{(n-1)/2}) + f(v_{(n-5)/2}) + 1$
- Define $f(v_{(n+1)/2}) = f(v_{(n-3)/2}) + 1$
- For $i = 1, 2, 3, 4......(n-5)/4$
  Define $f(V_{(n-1)/2-(2i+1)}) = f(v_{(n-1)/2-(2i-1)}) + f(v_{(n-1)/2-(2i-3)}) + 1$
- For $i = 1, 2, 3, 4......(n-13)/4$
  Define $f(V_{(n-1)/2+2i}) = f(v_{(n-1)/2-2i}) - (-1)^i$
- For $i = 1, 2, 3, 4......(n-5)/4$
  Define $f(V_{(n-1)/2+(2i+1)}) = f(v_{(n-1)/2-(2i+1)}) + (-1)^i$
- Define $f(v_{n-1}) = f(v_1) + f(v_3)$
  $f(v_n) = f(v_{n-1}) + 1$

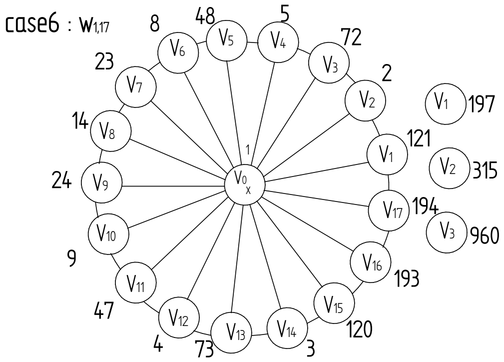

FIGURE 12. $on(W_{1,17}) = 3$

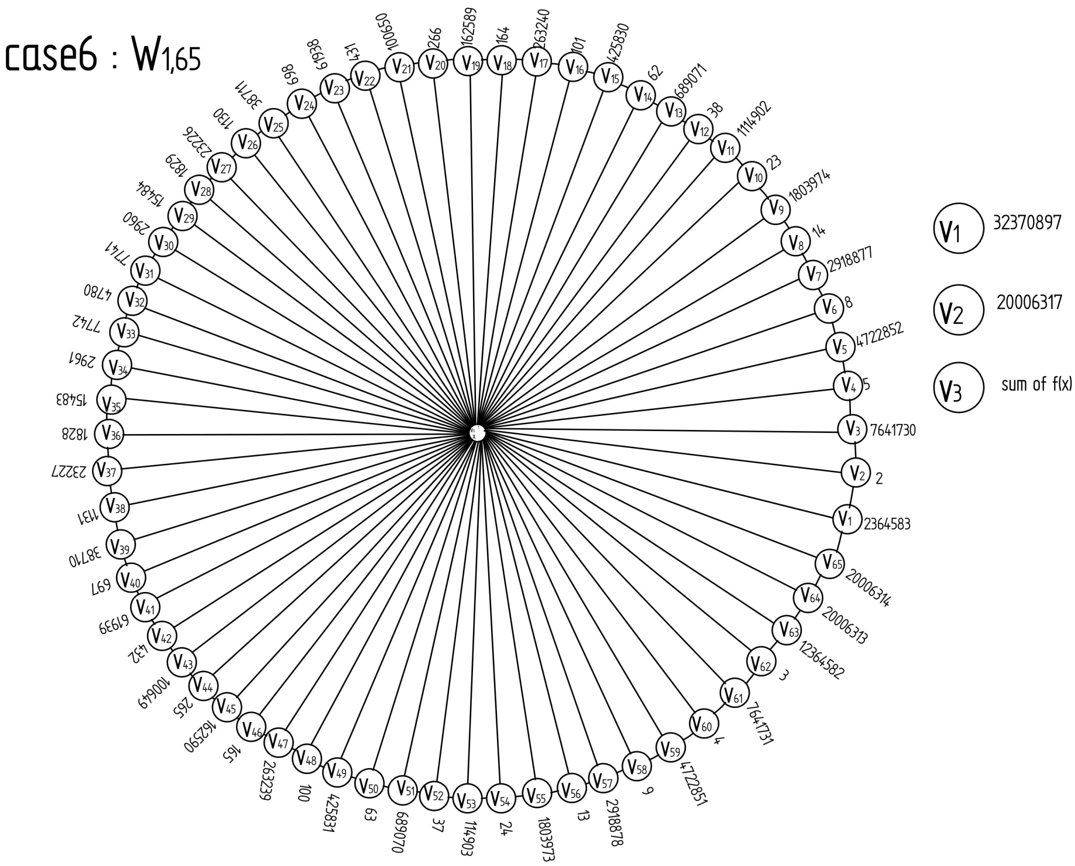

FIGURE 13. $on(W_{1,65}) = 3$

Case 7 : When $n = 18, 26, 34, 42, 50, ...$

- Define $f(v_2) = 2, f(v_4) = 5, f(v_{n-3}) = 3, f(v_{n-5}) = 4, f(x) = 1$
- For $i = 3, 4, 5......(n-2)/4$
  Define $f(V_{2i}) = f(v_{2i-4}) + f(v_{2i-2}) + 1$
- Define $f(v_{n/2}) = f(v_{(n-2)/2}) + 1$
- Define $f(v_{(n-4)/2}) = f(v_{(n-6)/2}) + f(v_{(n-2)/2}) + 1$
- Define $f(v_{(n-8)/2}) = f(v_{(n-4)/2}) + f(v_{n/2}) + 2$
- For $i = 2, 3, 4......(n-6)/4$
  Define $f(V_{(n-2)/2-(2i+1)}) = f(v_{(n-2)/2-(2i-1)}) + f(v_{(n-2)/2-(2i-3)}) + 1$
- For $i = 1, 2, 3, 4......(n-2)/4$
  Define $f(V_{n/2+(2i-1)}) = f(v_{n/2-2i}) - (-1)^i$
- For $i = 1, 2, 3, 4......(n-14)/4$
  Define $f(V_{n/2+2i}) = f(v_{n/2-(2i+1)}) - (-1)^i$
- Define $f(v_{n-1}) = f(v_1) + f(v_3)$
  $f(v_n) = f(v_1) + f(v_3) + 1$

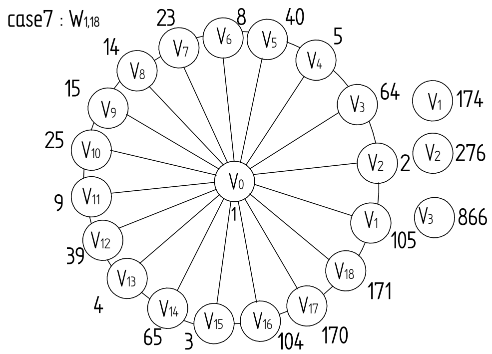

FIGURE 14. $on(W_{1,18}) = 3$

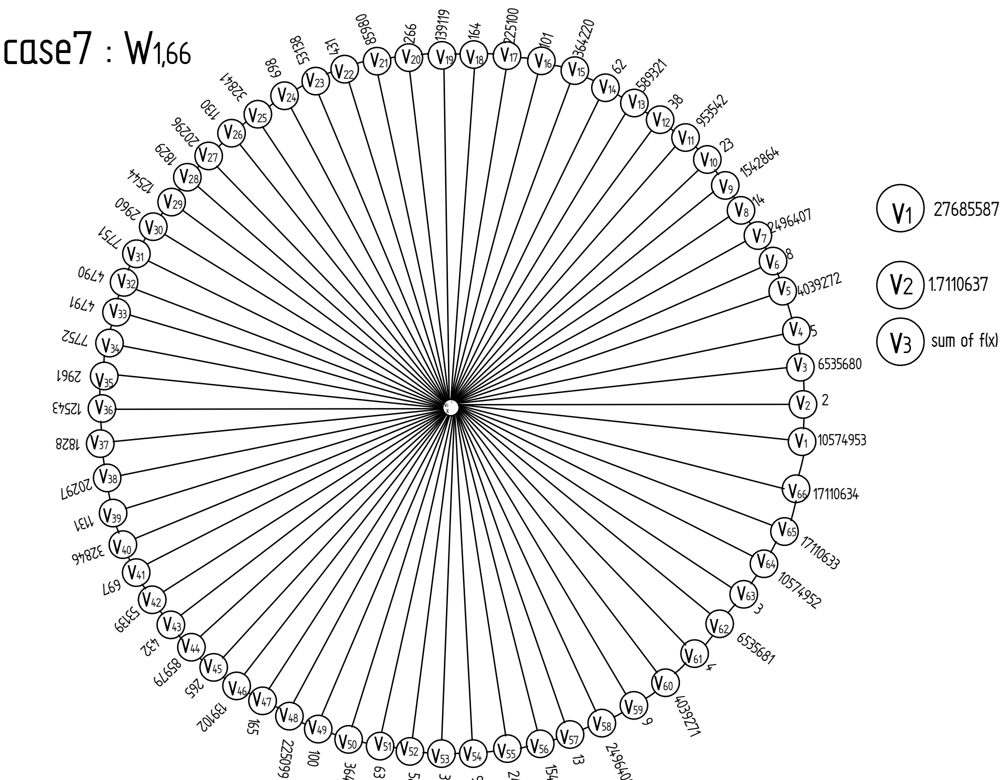

FIGURE 15. $on(W_{1,66}) = 3$

Case 8 : When $n = 19, 27, 35, 43, 51, ...$

- Define $f(v_2) = 2, f(v_4) = 5, f(v_{n-3}) = 3, f(v_{n-5}) = 4, f(x) = 1$
- For $i = 3, 4, 5......(n-7)/4$
  Define $f(V_{2i}) = f(v_{2i-4}) + f(v_{2i-2}) + 1$
- Define $f(v_{(n-3)/2}) = f(v_{(n-11)/2}) + f(v_{(n-7)/2}) + 1$
- Define $f(v_{(n-1)/2}) = f(v_{(n-7)/2}) + f(v_{(n-3)/2}) + 1$
- Define $f(v_{(n+1)/2}) = f(v_{(n-3)/2}) - 1$
- Define $f(v_{(n-5)/2}) = f(v_{(n-3)/2}) + f(v_{(n+1)/2})$
- For $i = 1, 2, 3, 4, 5......(n-7)/4$
  Define $f(V_{(n-5)/2-2i}) = f(v_{(n-5)/2-(2i-2)}) + f(v_{(n-5)/2+(2i-4)}) + 1 + [1-(-1)^i]/2$
- For $i = 1, 2, 3, 4......(n-3)/4$
  Define $f(V_{(n-1)/2+2i}) = f(v_{(n-1)/2-2i}) - (-1)^i$
- For $i = 1, 2, 3, 4......(n-15)/4$
  Define $f(V_{(n-1)/2+(2i+1)}) = f(v_{(n-1)/2-(2i+1)}) - (-1)^i$
- Define $f(v_{n-1}) = f(v_1) + f(v_3)$
  $f(v_n) = f(v_{n-1}) + 1$

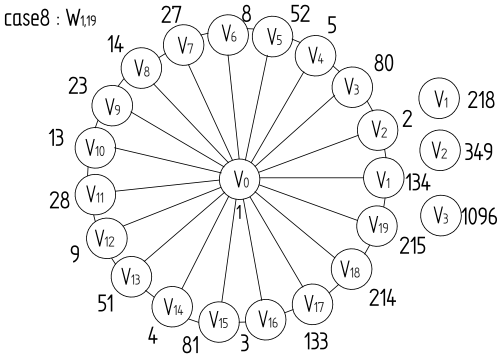

FIGURE 16. $on(W_{1,19}) = 3$

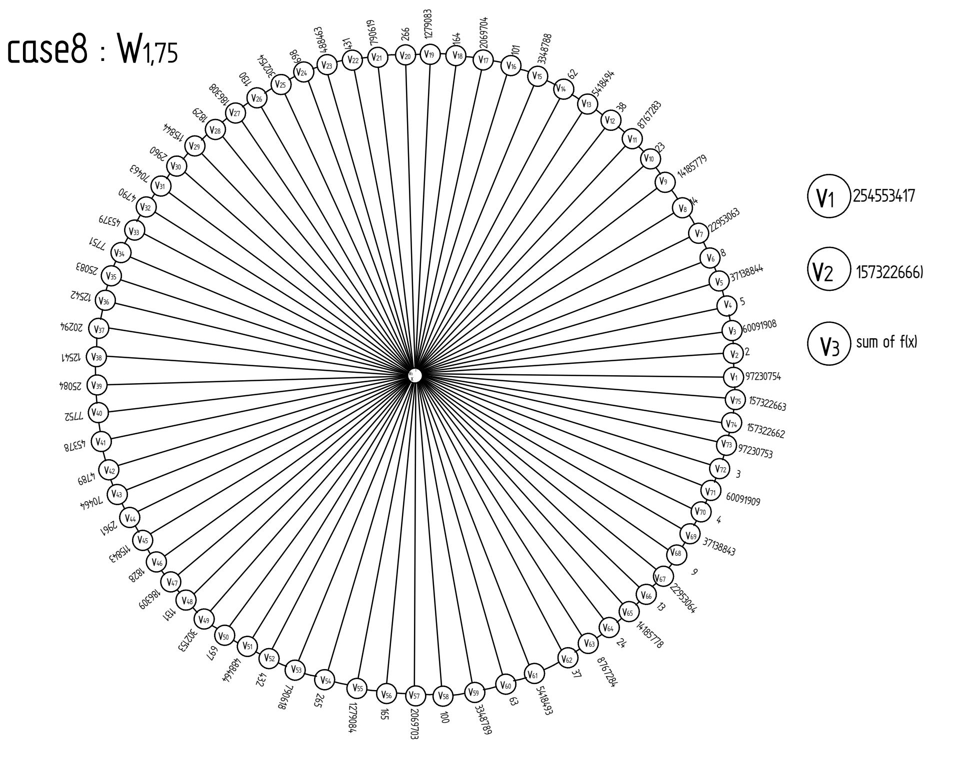

FIGURE 17. $on(W_{1,75}) = 3$

The function f defined in the above eight cases is clearly an injective function. Further, f is an outer sum labeling of $w_{1,n} \cup 3k_1$. Hence the proof.

ACKNOWLEDGMENT

The authors would like to thank the referees for their valuable suggestions and very much thankful to the Management of Reva University,Rukmini Knowledge Park,Kattigenahalli,yelahanka Bengaluru-64.

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