EFFECTS OF UNIFORM AND NON-UNIFORM TEMPERATURE GRADIENTS ON NON-DARCIAN-BENARD-MAGNETO-MARANGONI CONVECTION IN COMPOSITE LAYER IN THE PRESENCE OF CONSTANT HEAT SOURCE/SINK

MANJUNATHA N[1*] and SUMITHRA R [2]

[1]School of Applied Sciences, REVA University, Bengaluru, Karnataka, India;
Email: manjunatha.n@reva.edu.in

[2] Department of UG, PG Studies & Research in Mathematics, Government Science College Autonomous, Bengaluru, Karnataka, India;
Email: sumitra_diya@yahoo.com

ABSTRACT. ***The problem of Non-Darcian-Bènard-Magneto-Marangoni convection is considered in a composite layer which is horizontally boundless, in the presence of uniform heat source/sink in both the layers enclosed by isothermal boundaries. This composite layer is subjected to uniform and nonuniform temperature gradients and the eigenvalue, the thermal Marangoni numbers are obtained in the closed form for lower rigid and upper free with surface tension velocity conditions. The influence of horizontal wavenumber, porous parameter, Chandrasekhar number, viscosity ratio and the internal Rayleigh number on the stability of the system are discussed in detail. The effects of porous parameter and the vertical magnetic field are relatively interesting.***

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1. INTRODUCTION

Marangoni convective instabilities in composite layers the presence of nonuniform temperature gradients occur in many situations particularly in geophysics, astrophysics and some industrial problems like the manufacture of crystals and is not given much attention. Some of the work is available pertaining to Marangoni convection in presence of non uniform temperature gradients on single fluid / porous layers by Siddheshwar and Pranesh [23], Shivakumara *et al.* [22], Char and Chen [3], Ko-Ta Chiang [15] and Idris *et al.* [7]. Isa *et al.* [8, 9, 10] have investigated the effect of magnetic field on the onset of Marangoni and Bènard-Marangoni convection in a horizontal layer with a free-slip bottom heated from below and cooled from above with

*Corresponding author

non-uniform basic temperature gradient is considered. Melvian Johnson Fu *et al.* [16] have thoroughly investigated the effect of a non-uniform basic temperature gradient and magnetic field on the onset of Marangoni convection in a horizontal micropolar fluid layer bounded below by a rigid plate and above by non-deformable free surface subjected to a constant heat flux by using Rayleigh-Ritz technique. Chaoyang Zhang *et al.* [2] have considered Mesoscale simulation of Marangoni convection around a 2-D vapor bubble in a liquid, subjected to an imposed negative temperature gradient in the vertical direction under microgravity conditions is simulated with a newly developed Lattice Boltzmann liquid/vapor phase-change method. Sumithra and Manjunatha [25, 26] examined the effects of parabolic and inverted parabolic temperature gradients on surface tension driven magneto convection and double diffusive Marangoni convection in a composite layer in the closed form respectively.

The convective instabilities in the presence of heat source/sink has been widely studied by Rionero and Straughan [21], Rao and Wang [19], Rees and Pop [20], Parthiban and Patil [18], Khalili and Shivakumara [13], Herron [6], Khalili *et al.* [14], Joshi *et al.* [12], Vadasz[29], Nouri-Borujerdi *et al.* [17], Grosnan *et al.* [5], Cookey *et al.* [4] and Jawdat and Hashim [11]. Bhadauria [1] investigated double diffusive convection in a saturated anisotropic porous layer with internal heat source. Siddheshwar and Vanishree [24] have obtained Lorenz and Ginzburg Landau equations for thermal convection in a high porosity medium with heat source. Sumithra *et al.* [27] demonstrated the single component Marangoni convection is investigated in a composite layer, comprising of an incompressible single component fluid saturated porous layer over which lies a layer of same fluid with constant heat sources in both the layers. Sumithra and Manjunatha [28] discussed the problem of Bènard-Magneto-Marangoni convection in a composite layer which is horizontally infinite, is investigated for the Darcian case in the presence of constant heat source/sink in both the layers. One can look into the references there in for earlier works on the convective instabilities in the presence of heat source/sink.

The convective instabilities on composite layers and the same in the presence of additional heat source/sink and presence of temperature gradients is rarely found. In this direction, the problem of Non-Darcian-Bènard-Magneto Marangoni convection is considered in a composite layer which is horizontally boundless, in the presence of uniform heat source/sink in both the layers enclosed by isothermal boundaries. This composite layer is subjected to uniform and nonuniform temperature gradients and the eigenvalue, the thermal Marangoni numbers are obtained in the closed form for lower rigid and upper free with surface tension velocity conditions. The influence of horizontal wavenumber, porous parameter, Chandrasekhar number, viscosity ratio and the internal Rayleigh number on the stability of the system are discussed in detail.

2. Formulation of the problem

Consider a horizontal single component, electrically conducting fluid saturated isotropic sparsely packed porous layer of thickness d_m underlying a single component fluid layer of thickness d with an imposed magnetic field intensity H_0 in the vertical z- direction and with heat sources Φ_m and Φ respectively. The lower surface of the porous layer rigid and the upper surface of the fluid layer is free with surface tension effects depending on temperature. A Cartesian coordinate system is chosen with the origin at the interface between porous and fluid layers and the z-axis, vertically upwards. The basic equations for fluid and porous layer respectively governing such a system are,

$$\nabla.\overrightarrow{q} = 0 \tag{1}$$

$$\nabla.\overrightarrow{H} = 0 \tag{2}$$

$$\rho_0[\frac{\partial \overrightarrow{q}}{\partial t} + (\overrightarrow{q}.\nabla)\overrightarrow{q}] = -\nabla P + \mu\nabla^2\overrightarrow{q} + \mu_p(\overrightarrow{H}.\nabla)\overrightarrow{H} \tag{3}$$

$$\frac{\partial T}{\partial t} + (\overrightarrow{q}.\nabla)T = \kappa\nabla^2 T + \Phi \tag{4}$$

$$\frac{\partial \overrightarrow{H}}{\partial t} = \nabla \times \overrightarrow{q} \times \overrightarrow{H} + \nu_m\nabla^2\overrightarrow{H} \tag{5}$$

$$\nabla_m.\overrightarrow{q_m} = 0 \tag{6}$$

$$\nabla_m.\overrightarrow{H} = 0 \tag{7}$$

$$\rho_0[\frac{1}{\varepsilon}\frac{\partial \overrightarrow{q_m}}{\partial t} + \frac{1}{\varepsilon^2}(\overrightarrow{q_m}.\nabla_m)\overrightarrow{q_m}] = -\nabla_m P_m - \frac{\mu}{K}\overrightarrow{q_m} + \mu_m\nabla_m^2\overrightarrow{q_m} + \mu_p(\overrightarrow{H}.\nabla_m)\overrightarrow{H} \tag{8}$$

$$A\frac{\partial T_m}{\partial t} + (\overrightarrow{q_m}.\nabla_m)T_m = \kappa_m\nabla_m^2 T_m + \Phi_m \tag{9}$$

$$\varepsilon\frac{\partial \overrightarrow{H}}{\partial t} = \nabla_m \times \overrightarrow{q_m} \times \overrightarrow{H} + \nu_{em}\nabla_m^2\overrightarrow{H} \tag{10}$$

Where for fluid layer, $\overrightarrow{q}$ is the velocity vector, ρ_0 is the fluid density, t is time, μ is fluid viscosity, $P = p + \frac{\mu_p H^2}{2}$ is the total pressure, $\overrightarrow{H}$ is the magnetic field, T is temperature, constant heat source Φ , κ thermal diffusivity of the fluid, $\nu_m = \frac{1}{\mu_p\sigma}$ is the magnetic viscosity and μ_p is the magnetic permeability . For porous layer, ε is the porosity , μ_m is the effective viscosity of the fluid in the porous layer, K permeability of the porous medium, $A = \frac{(\rho_0 C_p)_m}{(\rho_0 C_p)_f}$ ratio of heat capacities, C_p specific heat, κ_m thermal diffusivity , Φ_m is constant heat source, $\nu_{em} = \frac{\nu_m}{\varepsilon}$ is the effective magnetic viscosity and the subscripts $'m'$ and $'f'$(in these equations) denotes the quantities in porous layer and fluid layer respectively.

The aim of this paper is to investigate the stability of a quiescent state to infinitesimal perturbations superposed on the basic state. The basic state of

the liquid being quiescent is described by

$$\overrightarrow{q} = \overrightarrow{q_b} = 0, P = P_b(z), T = T_b(z), \overrightarrow{H} = H_0(z) \tag{11}$$

$$\overrightarrow{q_m} = \overrightarrow{q_{mb}}, P_m = P_{mb}(z_m), T_m = T_{mb}(z_m), \overrightarrow{H} = H_0(z_m) \tag{12}$$

The basic state temperatures of $T_b(z)$ and $T_{mb}(z_m)$ are obtained as

$$T_b(z) = \frac{-\Phi z(z-d)}{2\kappa} + \frac{(T_u - T_0)h(z)}{d} + T_0 \quad 0 \le z \le d \tag{13}$$

$$T_{mb}(z_m) = \frac{-\Phi_m z_m(z_m + d_m)}{2\kappa_m} + \frac{(T_0 - T_l)h_m(z_m)}{d_m} + T_0$$
$$-d_m \le z_m \le 0 \tag{14}$$

where $T_0 = \dfrac{\kappa d_m T_u + \kappa_m d T_l}{\kappa d_m + \kappa_m d} + \dfrac{d d_m(\Phi_m d_m + \Phi d)}{2(\kappa d_m + \kappa_m d)}$ is the interface temperature and $h(z)$ and $h_m(z_m)$ are temperature gradients in fluid and porous layer respectively and subscript $'b'$ denote the basic state.
We superimpose infinitesimal disturbances on the basic state for fluid and porous layer respectively in the form

$$\overrightarrow{q} = \overrightarrow{q_b} + \overrightarrow{q}', P = P_b + P', T = T_b(z) + \theta, \overrightarrow{H} = H_0(z) + \overrightarrow{H'} \tag{15}$$

$$\overrightarrow{q_m} = \overrightarrow{q_{mb}} + \overrightarrow{q_m}', P_m = P_{mb} + P'_m, T_m = T_{mb}(z_m) + \theta_m,$$
$$\overrightarrow{H} = H_0(z_m) + \overrightarrow{H'} \tag{16}$$

where the prime indicates the perturbations. Introducing (11) and (12) in (1) - (10), operating curl twice and eliminate the pressure term from equations (3) and (8), the resulting equations then nondimensionalized.

The dimensionless equations are then subjected to normal mode analysis as follows

$$\begin{bmatrix} W \\ \theta \\ H \end{bmatrix} = \begin{bmatrix} W(z) \\ \theta(z) \\ H(z) \end{bmatrix} f(x,y)e^{nt} \tag{17}$$

$$\begin{bmatrix} W_m \\ \theta_m \\ H \end{bmatrix} = \begin{bmatrix} W_m(z_m) \\ \theta_m(z_m) \\ H(z_m) \end{bmatrix} f_m(x_m, y_m)e^{n_m t} \tag{18}$$

with $\nabla_2^2 f + a^2 f = 0$ and $\nabla_{2m}^2 f_m + a_m^2 f_m = 0$, where a and a_m are the horizontal wave numbers, n and n_m are the frequencies, $W(z)$ and $W_m(z_m)$ are the dimensionless vertical velocities in fluid and porous layer respectively and obtain the following equations
in $0 \le z \le 1$

$$(D^2 - a^2 + \frac{n}{Pr})(D^2 - a^2)W(z) = -Q\tau_{fm}D(D^2 - a^2)H(z) \tag{19}$$

$$(D^2 - a^2 + n)\theta(z) + [h(z) + R_I^*(2z-1)]W(z) = 0 \tag{20}$$

$$(\tau_{fm}(D^2 - a^2) + n)H(z) + DW(z) = 0 \tag{21}$$

in $-1 \leq z_m \leq 0$

$$[(D_m^2 - a_m^2)\widehat{\mu}\beta^2 + \frac{n_m\beta^2}{Pr_m} - 1](D_m^2 - a_m^2)W_m(z_m)$$
$$= -Q_m\tau_{mm}\beta^2 D_m(D_m^2 - a_m^2)H(z_m) \quad (22)$$
$$(D_m^2 - a_m^2 + An_m)\theta_m(z_m) + [h_m(z_m) + R_{Im}^*(2z_m + 1)]W_m(z_m) = 0 \quad (23)$$
$$\tau_{mm}(D_m^2 - a_m^2 + n_m\varepsilon)H(z_m) + D_m W_m(z_m) = 0 \quad (24)$$

Here for fluid layer, $Pr = \frac{\nu}{\kappa}$ is the prandtl number, $Q = \frac{\mu_p H_0^2 d^2}{\mu\kappa\tau_{fm}}$ is the Chandrasekhar number, $\tau_{fm} = \frac{\nu_{mv}}{\kappa}$ is the diffusivity ratio. For porous layer, $Pr_m = \frac{\varepsilon\nu_m}{\kappa_m}$ is the prandtl number, $Q_m = \frac{\mu_p H_0^2 d_m^2}{\mu\kappa_m\tau_{mm}} = Q\varepsilon\widehat{d^2}$ is the Chandrasekhar number, $\widehat{\mu} = \frac{\mu_m}{\mu}$ is the viscosity ratio, $\beta = \sqrt{\frac{K}{d_m^2}}$ is the porous parameter, $\tau_{mm} = \frac{\nu_{em}}{\kappa_m}$ is the diffusivity ratio. $R_I^* = \frac{R_I}{2(T_0 - T_u)}$, $R_{Im}^* = \frac{R_{Im}}{2(T_l - T_0)}$, here R_I is the internal Rayleigh number for fluid layer and R_{Im} is the internal Rayleigh number for porous layer. $h(z)$ and $h_m(z_m)$ are the non-dimensional temperature gradients with $\int_0^1 h(z)dz = 1$ and $\int_0^1 h_m(z_m)dz_m = 1$, $\theta(z)$ and $\theta_m(z_m)$ are the temperature in fluid and porous layers respectively.
Substituting the equation (21) in (19) and (24) in (22) to eliminate the magnetic field and assume that the present problem is satisfies the principle of exchange of stability, so putting $n = n_m = 0$. We get,

in $0 \leq z \leq 1$

$$(D^2 - a^2)^2 W(z) = QD^2 W(z) \quad (25)$$
$$(D^2 - a^2)\theta(z) + [h(z) + R_I^*(2z - 1)]W(z) = 0 \quad (26)$$

in $-1 \leq z_m \leq 0$

$$[(D_m^2 - a_m^2)\widehat{\mu}\beta^2 - 1](D_m^2 - a_m^2)W_m(z_m) = Q_m\beta^2 D_m^2 W_m(z_m) \quad (27)$$
$$(D_m^2 - a_m^2)\theta_m(z_m) + [h_m(z_m) + R_{Im}^*(2z_m + 1)]W_m(z_m) = 0 \quad (28)$$

3. Boundary Conditions

The boundary conditions are nondimensionalized and then subjected to normal mode expansion and are

$$\begin{aligned} D^2W(1) + Ma^2\theta(1) = 0, \\ W(1) = 0, W_m(-1) = 0, D_mW_m(-1) = 0, \\ \widehat{T}\widehat{d}^2(D^2 + a^2)W(0) = \widehat{\mu}(D_m^2 + a_m^2)W_m(0), \\ \widehat{T}W(0) = W_m(0), \widehat{T}\widehat{d}DW(0) = D_mW_m(0), \\ \widehat{T}\widehat{d}^3\beta^2(D^3W(0) - 3a^2DW(0)) = -D_mW_m(0) \\ +\widehat{\mu}\beta^2(D_m^3W_m(0) - 3a_m^2D_mW_m(0)), \\ \theta(1) = 0, \theta(0) = \widehat{T}\theta_m(0), \\ D\theta(0) = D_m\theta_m(0), \theta_m(-1) = 0 \end{aligned} \tag{29}$$

where
$\widehat{T} = \dfrac{T_l - T_0}{T_0 - T_u}$ is the thermal ratio, $M = -\dfrac{\partial\sigma_t}{\partial T}\dfrac{(T_0 - T_u)d}{\mu\kappa}$ is the thermal Marangoni number and $\widehat{d} = \dfrac{d_m}{d}$ is the depth ratio.

4. Method of Solution

The solutions $W(z)$ and $W_m(z_m)$ are obtained by solving (25) and (27) using the velocity boundary conditions of (29) we get,

$$W(z) = A_1[\cosh\delta z + a_1\sinh\delta z + a_2\cosh\zeta z + a_3\sinh\zeta z] \tag{30}$$

$$\begin{aligned} W_m(z_m) = A_1[a_4\cosh\eta_m z_m + a_5\sinh\eta_m z_m \\ +a_6\cosh\psi_m z_m + a_7\sinh\psi_m z_m] \end{aligned} \tag{31}$$

where
$\delta = \dfrac{\sqrt{Q} + \sqrt{Q + 4a^2}}{2}, \zeta = \dfrac{\sqrt{Q} - \sqrt{Q + 4a^2}}{2},$
$\eta_m = \sqrt{\dfrac{E + \sqrt{E^2 - 4F}}{2}}, \psi_m = \sqrt{\dfrac{E - \sqrt{E^2 - 4F}}{2}},$
$E = \dfrac{(2\widehat{\mu}\beta^2a_m^2 + 1 + Q_m\beta^2)}{\widehat{\mu}\beta^2}, F = \dfrac{(a_m^2 + a_m^4\widehat{\mu}\beta^2)}{\widehat{\mu}\beta^2},$
$a_1 = \frac{1}{\delta_{14}}(a_6\delta_{15} + a_7\delta_{16} + \delta_{17}), a_2 = a_6\delta_5 + \delta_6, a_3 = \frac{1}{\delta_9}(a_1\delta_{10} + a_7\delta_{11}),$
$a_4 = \delta_7 + a_6\delta_8, a_5 = a_1\delta_{12} + a_7\delta_{13}, a_6 = \dfrac{\delta_{23}\delta_{25} - \delta_{26}\delta_{22}}{\delta_{25}\delta_{21} - \delta_{24}\delta_{22}},$
$a_7 = \dfrac{\delta_{23}\delta_{24} - \delta_{26}\delta_{21}}{\delta_{24}\delta_{22} - \delta_{25}\delta_{21}}, \delta_1 = \widehat{T}\beta^2\widehat{d}^3(\delta^3 - 3a^2\delta), \delta_2 = \widehat{T}\beta^2\widehat{d}^3(\zeta^3 - 3a^2\zeta),$
$\delta_3 = \widehat{\mu}\beta^2(\eta_m^3 - 3a_m^2\eta_m) - \eta_m, \delta_4 = \widehat{\mu}\beta^2(\psi_m^3 - 3a_m^2\psi_m) - \psi_m,$
$\delta_5 = \dfrac{\widehat{\mu}[(\psi_m^2 + a_m^2) - \widehat{T}(\eta_m^2 + a_m^2)]}{[\widehat{T}\widehat{d}^2(\zeta^2 + a^2) - \widehat{\mu}\widehat{T}(\eta_m^2 + a_m^2)]},$

$$\delta_6 = \frac{[\widehat{\mu}(\eta_m^2 + a_m^2) - \widehat{d}^2(\delta^2 + a^2)]}{[\widehat{d}^2(\zeta^2 + a^2) - \widehat{\mu}(\eta_m^2 + a_m^2)]},$$
$\delta_7 = \widehat{T}(1+\delta_6), \delta_8 = \widehat{T}\delta_5 - 1, \delta_9 = \delta_2 - \frac{\widehat{T}\widehat{d}\zeta\delta_3}{\eta_m}, \delta_{10} = -\delta_1 + \frac{\widehat{T}\widehat{d}\delta\delta_3}{\eta_m},$
$\delta_{11} = \delta_4 - \frac{\psi\delta_3}{\eta_m}, \delta_{12} = \frac{1}{\eta_m}(\widehat{T}\widehat{d}\delta + \frac{\zeta\delta_{10}}{\delta_9}), \delta_{13} = \frac{1}{\eta_m}[(\frac{\widehat{T}\widehat{d}\zeta\delta_{11}}{\delta_9} - \psi_m)],$
$\delta_{14} = \sinh\delta + \frac{\delta_{10}\sinh\zeta}{\delta_9}, \delta_{15} = \delta_5\cosh\zeta, \delta_{16} = \frac{\delta_{11}\sinh\zeta}{\delta_9},$
$\delta_{17} = \delta_6\cosh\zeta + \cosh\delta, \delta_{18} = \frac{\delta_{12}\delta_{15}}{\delta_{14}}, \delta_{19} = \frac{\delta_{12}\delta_{16}}{\delta_{14}} + \delta_{13}, \delta_{20} = \frac{\delta_{12}\delta_{17}}{\delta_{14}},$
$\delta_{21} = \delta_8\cosh\eta_m - \delta_{18}\sinh\eta_m + \cosh\psi_m,$
$\delta_{22} = -\delta_{19}\sinh\eta_m - \sinh\psi_m, \delta_{23} = \delta_{20}\sinh\eta_m - \delta_7\cosh\eta_m,$
$\delta_{24} = -\eta_m\delta_8\sinh\eta_m - \delta_{18}\eta_m\cosh\eta_m - \psi_m\sinh\psi_m,$
$\delta_{25} = -\eta_m\delta_{19}\cosh\eta_m + \psi_m\cosh\psi_m,$
$\delta_{26} = \delta_{20}\eta_m\cosh\eta_m + \delta_7\eta_m\sinh\eta_m$

4.1. **Linear temperature profile.**

Consider the linear profile

$$h(z) = 1 \quad and \quad h_m(z_m) = 1 \tag{32}$$

substituting equation (32) into (26) and (28), the temperature distributions $\theta(z)$ and $\theta_m(z_m)$ are obtained using the temperature boundary conditions, as follows

$$\theta(z) = A_1[c_1\cosh az + c_2\sinh az + g_1(z)] \tag{33}$$
$$\theta_m(z_m) = A_1[c_3\cosh a_m z_m + c_4\sinh a_m z_m + g_{m1}(z_m)] \tag{34}$$

where
$g_1(z) = A_1[\delta_{27} - \delta_{28} + \delta_{29} - \delta_{30}],$
$g_{m1}(z_m) = A_1[\delta_{31} - \delta_{32} + \delta_{33} - \delta_{34}]$
$$\delta_{27} = \frac{(E_2 z + E_1)}{(\delta^2 - a^2)}(\cosh\delta z + a_1\sinh\delta z)$$
$$\delta_{28} = \frac{2\delta E_2}{(\delta^2 - a^2)^2}(a_1\cosh\delta z + \sinh\delta z)$$
$$\delta_{29} = \frac{(E_2 z + E_1)}{(\zeta^2 - a^2)}(a_2\cosh\zeta z + a_3\sinh\zeta z)$$
$$\delta_{30} = \frac{2\zeta E_2}{(\zeta^2 - a^2)^2}(a_3\cosh\zeta z + a_2\sinh\zeta z)$$
$$\delta_{31} = \frac{(E_{2m} z_m + E_{1m})}{(\eta_m^2 - a_m^2)}(a_4\cosh\eta_m z_m + a_5\sinh\eta_m z_m)$$
$$\delta_{32} = \frac{2\eta_m E_{2m}}{(\eta_m^2 - a_m^2)^2}(a_5\cosh\eta_m z_m + a_4\sinh\eta_m z_m)$$
$$\delta_{33} = \frac{(E_{2m} z_m + E_{1m})}{(\psi_m^2 - a_m^2)}(a_6\cosh\psi_m z_m + a_7\sinh\psi_m z_m)$$
$$\delta_{34} = \frac{2\psi_m E_{2m}}{(\psi_m^2 - a_m^2)^2}(a_7\cosh\psi_m z_m + a_6\sinh\psi_m z_m)$$
$E_1 = R_I^* - 1, E_2 = -2R_I^*, E_{1m} = -(R_{Im}^* + 1), E_{2m} = -2R_{Im}^*$
$c_1 = c_3\widehat{T} + \Delta_2 - \Delta_3, c_2 = \frac{1}{a}(c_4 a_m + \Delta_4 - \Delta_5),$

$$c_3 = \frac{\Delta_8\Delta_{10} + \Delta_{11}\Delta_7}{\Delta_6\Delta_{10} + \Delta_9\Delta_7}, c_4 = \frac{\Delta_8\Delta_9 - \Delta_{11}\Delta_6}{-\Delta_7\Delta_9 + \Delta_{10}\Delta_6},$$
$$\Delta_1 = -[\delta_{35} - \delta_{36} + \delta_{37} - \delta_{38}],$$
$$\delta_{35} = \frac{(E_2 + E_1)}{(\delta^2 - a^2)}(\cosh\delta + a_1\sinh\delta),$$
$$\delta_{36} = [\frac{2\delta E_2}{(\delta^2 - a^2)^2}](a_1\cosh\delta + \sinh\delta),$$
$$\delta_{37} = \frac{(E_2 + E_1)}{(\zeta^2 - a^2)}(a_2\cosh\zeta + a_3\sinh\zeta),$$
$$\delta_{38} = [\frac{2\zeta E_2}{(\zeta^2 - a^2)^2}](a_3\cosh\zeta + a_2\sinh\zeta),$$
$$\Delta_2 = \widehat{T}[\frac{E_{1m}a_4}{(\eta_m^2 - a_m^2)} - \frac{2E_{2m}\eta_m a_5}{(\eta_m^2 - a_m^2)^2} + \frac{E_{1m}a_6}{(\psi_m^2 - a_m^2)} - \frac{2E_{2m}\psi_m a_7}{(\psi_m^2 - a_m^2)^2}]$$
$$\Delta_3 = \frac{E_1}{(\delta^2 - a^2)} - \frac{2\delta a_1 E_2}{(\delta^2 - a^2)^2} + \frac{a_2 E_1}{(\zeta^2 - a^2)} - \frac{2\zeta a_3 E_2}{(\zeta^2 - a^2)^2},$$
$$\Delta_4 = [\frac{E_{2m}}{(\eta_m^2 - a_m^2)} - \frac{2\eta_m^2 E_{2m}}{(\eta_m^2 - a_m^2)^2}]a_4 + \frac{\eta_m a_5 E_{1m}}{(\eta_m^2 - a_m^2)} + \Delta_{400}$$
$$\Delta_{400} = [\frac{E_{2m}}{(\psi_m^2 - a_m^2)} - \frac{2\psi_m^2 E_{2m}}{(\psi_m^2 - a_m^2)^2}]a_6 + \frac{\psi_m a_7 E_{1m}}{(\psi_m^2 - a_m^2)}$$
$$\Delta_5 = \frac{E_1\delta a_1 + E_2}{(\delta^2 - a^2)} - \frac{2E_2\delta^2}{(\delta^2 - a^2)^2} + \frac{E_1\zeta a_3 + E_2 a_2}{(\zeta^2 - a^2)} - \frac{2a_2 E_2\zeta^2}{(\zeta^2 - a^2)^2}$$
$$\Delta_6 = \cosh a_m, \Delta_7 = \sinh a_m,$$
$$\Delta_8 = -[\delta_{39} - \delta_{40} + \delta_{41} - \delta_{42}]$$
$$\delta_{39} = \frac{(E_{1m} - E_{2m})}{(\eta_m^2 - a_m^2)}(a_4\cosh\eta_m - a_5\sinh\eta_m)$$
$$\delta_{40} = \frac{2\eta_m E_{2m}}{(\eta_m^2 - a_m^2)^2}(a_5\cosh\eta_m - a_4\sinh\eta_m)$$
$$\delta_{41} = \frac{(E_{1m} - E_{2m})}{(\psi_m^2 - a_m^2)}(a_6\cosh\psi_m - a_7\sinh\psi_m)$$
$$\delta_{42} = \frac{2\psi_m E_{2m}}{(\psi_m^2 - a_m^2)^2}(a_7\cosh\psi_m - a_6\sinh\psi_m)$$
$$\Delta_9 = \widehat{T}\cosh a, \Delta_{10} = \frac{a_m\sinh a}{a},$$
$$\Delta_{11} = \Delta_1 - (\Delta_2 - \Delta_3)\cosh a - \tfrac{1}{a}(\Delta_4 - \Delta_5)\sinh a$$

From the boundary condition (29), we have

$$M = \frac{-D^2W(1)}{a^2\theta(1)}$$

The thermal Marangoni number for the linear temperature profile is as follows

$$M_1 = -\frac{[\delta^2(\cosh\delta + a_1\sinh\delta) + \zeta^2(a_2\cosh\zeta + a_3\sinh\zeta)]}{a^2(c_1\cosh a + c_2\sinh a + \Lambda_1 + \Lambda_2)} \tag{35}$$

where
$$\Lambda_1 = \frac{(E_2+E_1)}{(\delta^2-a^2)}(\cosh\delta + a_1\sinh\delta) - \frac{2\delta E_2}{(\delta^2-a^2)^2}(a_1\cosh\delta+\sinh\delta)$$
$$\Lambda_2 = \frac{(E_2+E_1)}{(\zeta^2-a^2)}(a_2\cosh\zeta + a_3\sinh\zeta) - \frac{2\zeta E_2}{(\zeta^2-a^2)^2}(a_3\cosh\zeta+a_2\sinh\zeta)$$

4.2. **Parabolic temperature profile.**

For the parabolic temperature profile
$$h(z) = 2z \quad and \quad h_m(z_m) = 2z_m \tag{36}$$
substituting (36) into (26) and (28), the temperature distributions $\theta(z)$ and $\theta_m(z_m)$ are obtained using the temperature boundary conditions is as follows
$$\theta(z) = A_1[c_5\cosh az + c_6\sinh az + g_2(z)] \tag{37}$$
$$\theta_m(z_m) = A_1[c_7\cosh a_m z_m + c_8\sinh a_m z_m + g_{m2}(z_m)] \tag{38}$$
where
$g_2(z) = A_1[\delta_{43}-\delta_{44}+\delta_{45}-\delta_{46}], g_{m2}(z_m) = A_1[\delta_{47}-\delta_{48}+\delta_{49}-\delta_{50}]$
$$\delta_{43} = \frac{(E_4 z+E_3)}{(\delta^2-a^2)}(\cosh\delta z + a_1\sinh\delta z)$$
$$\delta_{44} = \frac{2\delta E_4}{(\delta^2-a^2)^2}(a_1\cosh\delta z+\sinh\delta z)$$
$$\delta_{45} = \frac{(E_4 z+E_3)}{(\zeta^2-a^2)}(a_2\cosh\zeta z + a_3\sinh\zeta z)$$
$$\delta_{46} = \frac{2\zeta E_4}{(\zeta^2-a^2)^2}(a_3\cosh\zeta z+a_2\sinh\zeta z)$$
$$\delta_{47} = \frac{(E_{4m}z_m+E_{3m})}{(\eta_m^2-a_m^2)}(a_4\cosh\eta_m z_m + a_5\sinh\eta_m z_m)$$
$$\delta_{48} = \frac{2\eta_m E_{4m}}{(\eta_m^2-a_m^2)^2}(a_5\cosh\eta_m z_m+a_4\sinh\eta_m z_m)$$
$$\delta_{49} = \frac{(E_{4m}z_m+E_{3m})}{(\psi_m^2-a_m^2)}(a_6\cosh\psi_m z_m + a_7\sinh\psi_m z_m)$$
$$\delta_{50} = \frac{2\psi_m E_{4m}}{(\psi_m^2-a_m^2)^2}(a_7\cosh\psi_m z_m+a_6\sinh\psi_m z_m)$$
$E_3 = R_I^*, E_4 = -2(R_I^*+1), E_{3m} = -R_{Im}^*, E_{4m} = -2(R_{Im}^*+1)$
$c_5 = c_7\widehat{T} + \Delta_{13} - \Delta_{14}, c_6 = \frac{1}{a}(c_8 a_m + \Delta_{15} - \Delta_{16}),$
$$c_7 = \frac{\Delta_{19}\Delta_{21}+\Delta_{22}\Delta_{18}}{\Delta_{17}\Delta_{21}+\Delta_{20}\Delta_{18}}, c_8 = \frac{\Delta_{19}\Delta_{20}-\Delta_{22}\Delta_{17}}{-\Delta_{18}\Delta_{20}-\Delta_{21}\Delta_{17}},$$
$\Delta_{12} = -[\delta_{51}-\delta_{52}+\delta_{53}-\delta_{54}],$
$$\delta_{51} = \frac{(E_4+E_3)}{(\delta^2-a^2)}(\cosh\delta + a_1\sinh\delta),$$
$$\delta_{52} = [\frac{2\delta E_4}{(\delta^2-a^2)^2}](\cosh a_1\delta + \sinh\delta),$$

$$\delta_{53} = \frac{(E_4 + E_3)}{(\zeta^2 - a^2)}(a_2 \cosh\zeta + a_3 \sinh\zeta),$$
$$\delta_{54} = [\frac{2\zeta E_4}{(\zeta^2 - a^2)^2}](a_3 \cosh\zeta + a_2 \sinh\zeta),$$
$$\Delta_{13} = \widehat{T}[\frac{E_{3m}a_4}{(\eta_m^2 - a_m^2)} - \frac{2E_{4m}\eta_m a_5}{(\eta_m^2 - a_m^2)^2} + \frac{E_{3m}a_6}{(\psi_m^2 - a_m^2)} - \frac{2E_{4m}\psi_m a_7}{(\psi_m^2 - a_m^2)^2}],$$
$$\Delta_{14} = \frac{E_3}{(\delta^2 - a^2)} - \frac{2\delta a_1 E_4}{(\delta^2 - a^2)^2} + \frac{a_2 E_3}{(\zeta^2 - a^2)} - \frac{2\zeta a_3 E_4}{(\zeta^2 - a^2)^2},$$
$$\Delta_{15} = [\frac{E_{4m}}{(\eta_m^2 - a_m^2)} - \frac{2\eta_m^2 E_{4m}}{(\eta_m^2 - a_m^2)^2}]a_4 + \frac{\eta_m a_5 E_{3m}}{(\eta_m^2 - a_m^2)} + \Delta_{150}$$
$$\Delta_{150} = [\frac{E_{4m}}{(\psi_m^2 - a_m^2)} - \frac{2\psi_m^2 E_{4m}}{(\psi_m^2 - a_m^2)^2}]a_6 + \frac{\psi_m a_7 E_{3m}}{(\psi_m^2 - a_m^2)}$$
$$\Delta_{16} = \frac{E_3\delta a_1 + E_4}{(\delta^2 - a^2)} - \frac{2E_4\delta^2}{(\delta^2 - a^2)^2} + \frac{E_3\zeta a_3 + E_4 a_2}{(\zeta^2 - a^2)} - \frac{2a_2 E_4 \zeta^2}{(\zeta^2 - a^2)^2},$$
$$\Delta_{17} = \cosh a_m, \Delta_{18} = \sinh a_m,$$
$$\Delta_{19} = -[\delta_{55} - \delta_{56} + \delta_{57} - \delta_{58}]$$
$$\delta_{55} = \frac{(E_{3m} - E_{4m})}{(\eta_m^2 - a_m^2)}(a_4 \cosh\eta_m - a_5 \sinh\eta_m)$$
$$\delta_{56} = \frac{2\eta_m E_{4m}}{(\eta_m^2 - a_m^2)^2}(a_5 \cosh\eta_m - a_4 \sinh\eta_m)$$
$$\delta_{57} = \frac{(E_{3m} - E_{4m})}{(\psi_m^2 - a_m^2)}(a_6 \cosh\psi_m - a_7 \sinh\psi_m)$$
$$\delta_{58} = \frac{2\psi_m E_{4m}}{(\psi_m^2 - a_m^2)^2}(a_7 \cosh\psi_m - a_6 \sinh\psi_m)$$
$$\Delta_{20} = \widehat{T}a \sinh a, \Delta_{21} = a_m \cosh a,$$
$$\Delta_{22} = \Delta_{12} - a(\Delta_{13} - \Delta_{14}) \sinh a - (\Delta_{15} - \Delta_{16}) \cosh a$$

From the boundary condition (29), the thermal Marangoni number for parabolic temperature profile is as follows

$$M_2 = -\frac{[\delta^2(\cosh\delta + a_1 \sinh\delta) + \zeta^2(a_2 \cosh\zeta + a_3 \sinh\zeta)]}{a^2(c_5 \cosh a + c_6 \sinh a + \Lambda_3 + \Lambda_4)} \tag{39}$$

where

$$\Lambda_3 = \frac{(E_4 + E_3)}{(\delta^2 - a^2)}(\cosh\delta + a_1 \sinh\delta) - \frac{2\delta E_4}{(\delta^2 - a^2)^2}(a_1 \cosh\delta + \sinh\delta)$$
$$\Lambda_4 = \frac{(E_4 + E_3)}{(\zeta^2 - a^2)}(a_2 \cosh\zeta + a_3 \sinh\zeta) - \frac{2\zeta E_4}{(\zeta^2 - a^2)^2}(a_3 \cosh\zeta + a_2 \sinh\zeta)$$

4.3. **Inverted Parabolic temperature profile.**

Consider this profile

$$h(z) = 2(1 - z) \quad and \quad h_m(z_m) = 2(1 - z_m) \tag{40}$$

substituting (40) into (26) and (28), the temperature distributions $\theta(z)$ and $\theta_m(z_m)$ are obtained using the temperature boundary conditions, as follows

$$\theta(z) = A_1[c_9 \cosh az + c_{10} \sinh az + g_3(z)] \tag{41}$$

$$\theta_m(z_m) = A_1[c_{11} \cosh a_m z_m + c_{12} \sinh a_m z_m + g_{m3}(z_m)] \tag{42}$$

where

$g_3(z) = A_1[\delta_{59} - \delta_{60} + \delta_{61} - \delta_{62}], g_{m3}(z_m) = A_1[\delta_{63} - \delta_{64} + \delta_{65} - \delta_{66}]$

$\delta_{59} = \dfrac{(E_6 z + E_5)}{(\delta^2 - a^2)}(\cosh \delta z + a_1 \sinh \delta z)$

$\delta_{60} = \dfrac{2\delta E_6}{(\delta^2 - a^2)^2}(a_1 \cosh \delta z + \sinh \delta z)$

$\delta_{61} = \dfrac{(E_6 z + E_5)}{(\zeta^2 - a^2)}(a_2 \cosh \zeta z + a_3 \sinh \zeta z)$

$\delta_{62} = \dfrac{2\zeta E_6}{(\zeta^2 - a^2)^2}(a_3 \cosh \zeta z + a_2 \sinh \zeta z)$

$\delta_{63} = \dfrac{(E_{6m} z_m + E_{5m})}{(\eta_m^2 - a_m^2)}(a_4 \cosh \eta_m z_m + a_5 \sinh \eta_m z_m)$

$\delta_{64} = \dfrac{2\eta_m E_{6m}}{(\eta_m^2 - a_m^2)^2}(a_5 \cosh \eta_m z_m + a_4 \sinh \eta_m z_m)$

$\delta_{65} = \dfrac{(E_{6m} z_m + E_{5m})}{(\psi_m^2 - a_m^2)}(a_6 \cosh \psi_m z_m + a_7 \sinh \psi_m z_m)$

$\delta_{66} = \dfrac{2\psi_m E_{6m}}{(\psi_m^2 - a_m^2)^2}(a_7 \cosh \psi_m z_m + a_6 \sinh \psi_m z_m)$

$E_5 = R_I^* - 2, E_6 = 2(1 - R_I^*), E_{5m} = -2 - R_{Im}^*, E_{6m} = 2(1 - R_{Im}^*)$

$c_9 = c_{11}\widehat{T} + \Delta_{24} - \Delta_{25}, c_{10} = \frac{1}{a}(c_{12} a_m + \Delta_{26} - \Delta_{27}),$

$c_{11} = \dfrac{\Delta_{30}\Delta_{32} + \Delta_{33}\Delta_{29}}{\Delta_{28}\Delta_{32} + \Delta_{31}\Delta_{29}}, c_{12} = \dfrac{\Delta_{30}\Delta_{31} - \Delta_{33}\Delta_{28}}{-\Delta_{29}\Delta_{31} - \Delta_{32}\Delta_{28}},$

$\Delta_{23} = -[\delta_{67} - \delta_{68} + \delta_{69} - \delta_{70}],$

$\delta_{67} = \dfrac{(E_6 + E_5)}{(\delta^2 - a^2)}(\cosh \delta + a_1 \sinh \delta),$

$\delta_{68} = [\dfrac{2\delta E_6}{(\delta^2 - a^2)^2}](a_1 \cosh \delta + \sinh \delta),$

$\delta_{69} = \dfrac{(E_6 + E_5)}{(\zeta^2 - a^2)}(a_2 \cosh \zeta + a_3 \sinh \zeta),$

$\delta_{70} = [\dfrac{2\zeta E_6}{(\zeta^2 - a^2)^2}](a_3 \cosh \zeta + a_2 \sinh \zeta),$

$\Delta_{24} = \widehat{T}[\dfrac{E_{5m} a_4}{(\eta_m^2 - a_m^2)} - \dfrac{2E_{6m}\eta_m a_5}{(\eta_m^2 - a_m^2)^2} + \dfrac{E_{5m} a_6}{(\psi_m^2 - a_m^2)} - \dfrac{2E_{6m}\psi_m a_7}{(\psi_m^2 - a_m^2)^2}],$

$\Delta_{25} = \dfrac{E_5}{(\delta^2 - a^2)} - \dfrac{2\delta a_1 E_6}{(\delta^2 - a^2)^2} + \dfrac{a_2 E_5}{(\zeta^2 - a^2)} - \dfrac{2\zeta a_3 E_6}{(\zeta^2 - a^2)^2},$

$\Delta_{26} = [\dfrac{E_{6m}}{(\eta_m^2 - a_m^2)} - \dfrac{2\eta_m^2 E_{6m}}{(\eta_m^2 - a_m^2)^2}]a_4 + \dfrac{\eta_m a_5 E_{5m}}{(\eta_m^2 - a_m^2)} + \Delta_{260}$

$\Delta_{260} = [\dfrac{E_{6m}}{(\psi_m^2 - a_m^2)} - \dfrac{2\psi_m^2 E_{6m}}{(\psi_m^2 - a_m^2)^2}]a_6 + \dfrac{\psi_m a_7 E_{5m}}{(\psi_m^2 - a_m^2)}$

$$\Delta_{27} = \frac{E_5\delta a_1 + E_6}{(\delta^2 - a^2)} - \frac{2E_6\delta^2}{(\delta^2 - a^2)^2} + \frac{E_5\zeta a_3 + E_6 a_2}{(\zeta^2 - a^2)} - \frac{2a_2E_6\zeta^2}{(\zeta^2 - a^2)^2},$$
$$\Delta_{28} = \cosh a_m, \Delta_{29} = \sinh a_m,$$
$$\Delta_{30} = -[\delta_{71} - \delta_{72} + \delta_{73} - \delta_{74}]$$
$$\delta_{71} = \frac{(E_{5m} - E_{6m})}{(\eta_m^2 - a_m^2)}(a_4 \cosh \eta_m - a_5 \sinh \eta_m)$$
$$\delta_{72} = \frac{2\eta_m E_{6m}}{(\eta_m^2 - a_m^2)^2}(a_5 \cosh \eta_m - a_4 \sinh \eta_m)$$
$$\delta_{73} = \frac{(E_{5m} - E_{6m})}{(\psi_m^2 - a_m^2)}(a_6 \cosh \psi_m - a_7 \sinh \psi_m)$$
$$\delta_{74} = \frac{2\psi_m E_{6m}}{(\psi_m^2 - a_m^2)^2}(a_7 \cosh \psi_m - a_6 \sinh \psi_m)$$
$$\Delta_{31} = \widehat{T} \cosh a, \Delta_{32} = \frac{a_m \sinh a}{a},$$
$$\Delta_{33} = \Delta_{23} - (\Delta_{24} - \Delta_{25}) \cosh a - (\Delta_{26} - \Delta_{27}) \sinh a$$

From the boundary condition (29), the thermal Marangoni number for inverted parabolic temperature profile is as follows

$$M_3 = -\frac{[\delta^2(\cosh \delta + a_1 \sinh \delta) + \zeta^2(a_2 \cosh \zeta + a_3 \sinh \zeta)]}{a^2(c_9 \cosh a + c_{10} \sinh a + \Lambda_5 + \Lambda_6)} \tag{43}$$

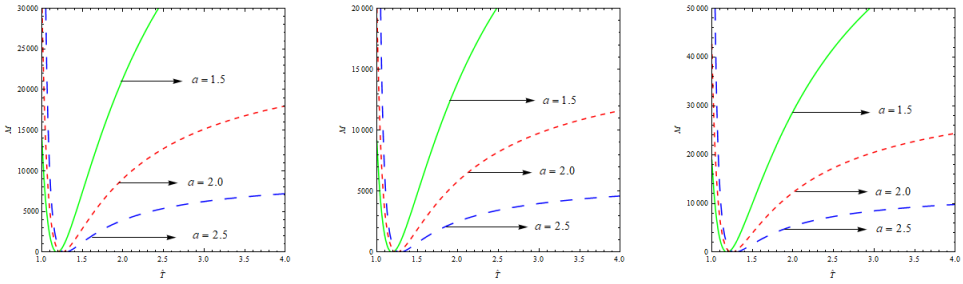
where

$$\Lambda_5 = \frac{(E_6 + E_5)}{(\delta^2 - a^2)}(\cosh \delta + a_1 \sinh \delta) - \frac{2\delta E_6}{(\delta^2 - a^2)^2}(a_1 \cosh \delta + \sinh \delta)$$
$$\Lambda_6 = \frac{(E_6 + E_5)}{(\zeta^2 - a^2)}(a_2 \cosh \zeta + a_3 \sinh \zeta) - \frac{2\zeta E_6}{(\zeta^2 - a^2)^2}(a_3 \cosh \zeta + a_2 \sinh \zeta)$$

5. Results and Discussion

The thermal Marangoni number M is obtained as an expression of the depth ratio $\widehat{d}$, the horizontal wave numbers a and a_m both for the fluid and porous layers, the porous parameter β, the thermal ratio $\widehat{T}$, the viscosity ratio $\widehat{\mu}$, R_I and R_{Im} the internal Rayleigh numbers for the fluid and porous layers and the Chandrasekhar number Q. This thermal Marangoni number M is drawn as a function of thermal ratio $\widehat{T}$ where the other parameters for fixed . From the graphs it is evident that, for smaller values of $\widehat{T}$, the thermal Marangoni number M decreases and the thermal Marangoni increases as the value of thermal ratio is further increased. The effects of the horizontal wave number a, the porous parameter β , the Chandrasekhar number Q, the viscosity ratio $\widehat{\mu}$ and the internal Rayleigh number R_I on the thermal Marangoni number are displayed in the following figures where the supplementary parameters are fixed and they are $Q = 10, \varepsilon = 1, \beta = 0.1, \widehat{d} = 2.5, a = 2.5, \widehat{\mu} = 2.5, R_I = -3$ and $R_{Im} = 1$.

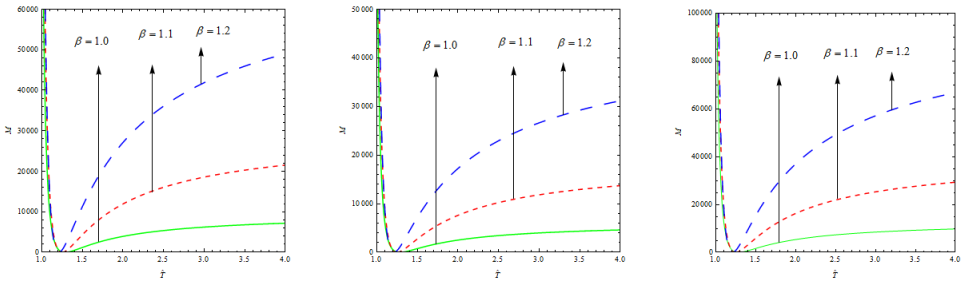
Fig.1 exhibits the effects of the horizontal wave number a on the values of thermal Marangoni number M for the values of $a = 1.5, 2.0$ and 2.5. From the figure it is clear that for smaller values of thermal ratio there is



(a) (b) (c)

FIGURE 1. Effects of horizontal wave number a

no effect of horizontal wavenumber a on thermal Marangoni number M for all the temperature profiles. For larger values of thermal ratio, there is drastic effect of this parameter on thermal Marangoni number. For a fixed value of thermal ratio, increase in the value of a, decreases the thermal Marangoni number. Hence the system becomes stable by decreasing the horizontal wave number. Similar effects are observed for both uniform and non uniform (parabolic and inverted parabolic) temperature profiles.

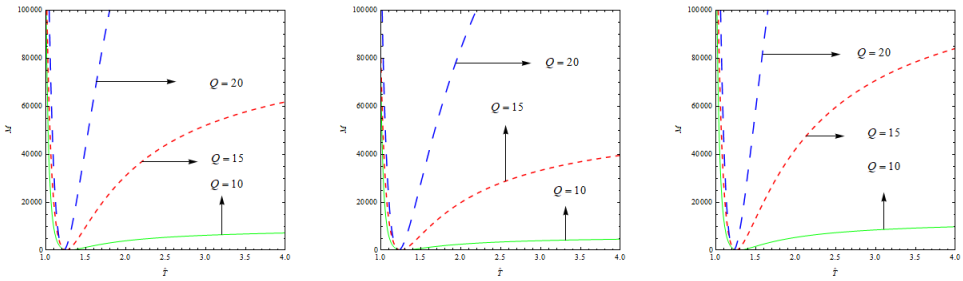


(a) (b) (c)

FIGURE 2. Effects of porous parameter β

Fig. 2 depicts the effects of β the porous parameter on the thermal Marangoni number and it is for $\beta = 1.1, 1.2$ and 1.3. The curves are diverging for all the temperature profiles, which indicates that the effect of the porous parameter is leading for larger values of thermal ratio. For fixed values of thermal ratio, the increase in the value of β, shows the increase in the Marangoni number. Hence the system can be stabilized by increasing the value of β. It is interesting to note that, though there is more permeability for the fluid, the system still remains stabilized, this may be due to presence of vertical Magnetic field and presence of heat source in the porous layer.

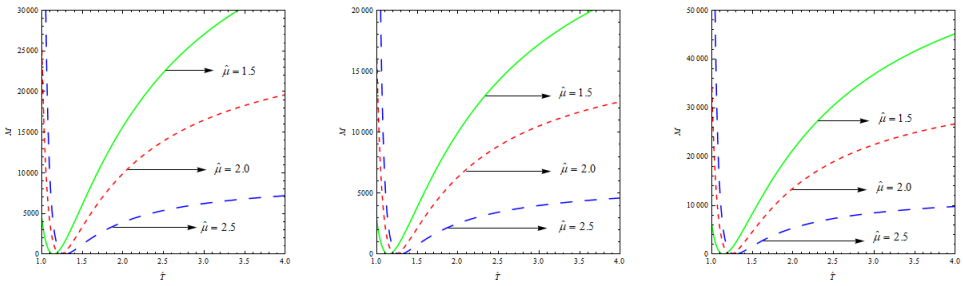
The effects of Chandrasekhar number Q is displayed in Fig. 3 for the linear, parabolic and inverted parabolic temperature profiles. The values of Q taken are $10, 15$ and 20. The curves are for all the three profiles are diverging drastically, indicating the prominence of Q for larger thermal ratio values.



(a) (b) (c)

FIGURE 3. Effects of Chandrasekhar number Q

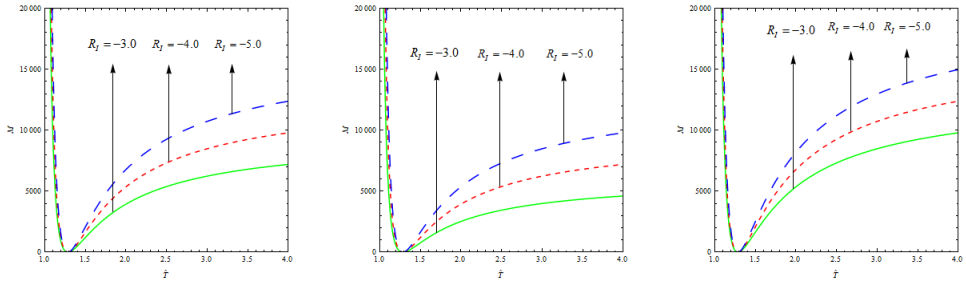
For a fixed thermal ratio, the increase in the value of Q increases the thermal Marangoni number, hence the Non-Darcy-Bènard-Magneto-Marangoni convection can be advanced by decreasing the values of Q and hence the system can be destabilized. The presence of magnetic field postpones the onset of Non-Darcy-Bènard-Marangoni convection.



(a) (b) (c)

FIGURE 4. Effects of viscosity ratio $\widehat{\mu}$

The effects of the viscosity ratio $\widehat{\mu}$ on the thermal Marangoni number is shown in the Fig. 4 for all the three temperature profile. The curves are diverging for larger values of thermal ratio, which indicates that the effect of $\widehat{\mu}$ is effective only for the larger values of thermal ratio and the increase in the value of viscosity ratio decreases the thermal Marangoni number M, hence, the system can be stabilized by decreasing the value of viscosity ratio $\widehat{\mu}$. So, the Non-Darcy-Bènard-Magneto-Marangoni convection can be postponed by reducing the value of viscosity ratio $\widehat{\mu}$.
The effect of internal Rayleigh number R_I on the Marangoni number is similar for all the three temperature profiles depicted the Fig. 5 for $R_I = -3, -4$ and -5. Decreasing the values of R_I, the Marangoni number increases, hence the Non-Darcy-Bènard-Magneto-Marangoni convection can be deferred by decreasing the values of R_I. That is heat absorption in the fluid layer stabilizes the composite layer system.



(a) (b) (c)

FIGURE 5. Effects of internal Rayleigh number R_I

6. CONCLUSION

Following conclusions are drawn from this study

(i) Reducing the values of porous parameter and Chandrasekhar number prepone Non-Darcy-Bènard-Magneto-Marangoni convection.
(ii) Reducing the values of the horizontal wavenumber a and viscosity ratio $\widehat{\mu}$ postpone Non-Darcy-Bènard-Magneto-Marangoni convection.
(iii) The presence of heat sink in the fluid layer postpones Non-Darcy-Bènard-Magneto-Marangoni convection.
(iv) There is no effect of internal Rayleigh number R_{Im} on the Non-Darcy-Bènard-Magneto-Marangoni convection.
(v) The inverted parabolic temperature gradient is the highly stable of all the three gradients.
(vi) The effects of the physical parameters is akin to both uniform and nonuniform temperature profiles.

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