# EFFECTS OF HEAT SOURCE/SINK AND NON UNIFORM TEMPERATURE GRADIENTS ON BENARD-SURFACE TENSION DRIVEN CONVECTION IN A COMPOSITE LAYER IN THE PRESENCE OF VERTICAL MAGNETIC FIELD

**MANJUNATHA N**[1*] and **SUMITHRA R** [2]

[1]School of Applied Sciences, REVA University, Bengaluru, Karnataka, India;
Email: manjunatha.n@reva.edu.in

[2] Department of UG, PG Studies & Research in Mathematics, Government Science College Autonomous, Bengaluru, Karnataka, India;
Email: sumitra_diya@yahoo.com

ABSTRACT. ***The problem of Bènard-Surface tension driven convection in a composite layer which is parallel infinitely long in x and y directions, is considered for the Darcian case in the presence of invariable heat source/sink in both the layers and vertical magnetic field. This composite layer is subjected to uniform and non uniform temperature gradients. The eigenvalue, thermal Marangoni number is obtained in closed form with the lower adiabatic surface rigid and upper isothermal surface free with surface tension effects for the velocity boundary combinations. The influence of various parameters on the eigenvalue against thermal ratio is discussed in detail. It is observed that the effect of heat source/sink in the fluid layer is dominant on the eigenvalue over the same in the porous layer. The important parameters that operate (advance or delay) surface tension driven convection are determined.***

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## 1. INTRODUCTION

Many chemical engineering processes such as metallurgical and polymer extrusion processes involve cooling of molten liquid being stretched into a cooling system, the fluid mechanical properties of the penultimate product depend upon mainly on the cooling liquid used and the rate of stretching. Some polymer fluids such as polyethylene oxide and polyisobutylene solution in cetane, having better electromagnetic properties, are namely used as cooling liquid as their flow can be regulated by external magnetic fields in order to improve the quality of final product. The study of heat source/sink effects on heat transfer is very important because their effects are crucial in

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*Corresponding author

controlling the heat transfer also used as an effective parameter to control convection. The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Heat generation effects may alter the temperature distribution, and these in turn can affect the particle deposition rate in nuclear reactors, electronic chips, and semiconductors wafers.

Some literature is available on the single component, Balasubrahmanyam *et al.* [1] studied the Soret effect on mixed convective heat and mass transfer through a porous medium confined in a cylindrical annulus under a radial magnetic field in the presence of a constant heat source/sink. Thommaandru Ranga Rao *et al.* [16] investigates theoretically the problem of free convection boundary layer flow of nanofluids over a nonlinear stretching sheet in the presence of MHD and heat source/sink using fourth order Runge-Kutta method along with shooting technique. Dileep Kumar and Singh [2] investigated the effects of induced magnetic field and heat source/sink on fully developed laminar natural convective flow of a viscous incompressible and electrically conducting fluid in the presence of radial magnetic field by considering induced magnetic field into account. The governing equations of the considered model are transformed into simultaneously ordinary differential equations and solved analytically. Isa *et al.*[9, 10] studied the problem of Bènard-Marangoni convection in a horizontal fluid layer heated from below with non-uniform temperature gradient under magnetic field using linear stability analysis. The eigenvalues are obtained for both adiabatic boundaries also six non-uniform basic temperature profiles are considered. Using linear stability analysis, Mokhtar *et al.* [8] is applied to investigate the effect of internal heat generation on Marangoni convection in a two-layer system comprising an incompressible fluid-saturated porous layer over which lies a layer of the same fluid.

Recently, Using Laplace transformation technique, the effect of heat source / sink past an impulsively started vertical plate under the influence of transverse and uniform magnetic field has been investigated by Garg and Shipra [3, 4, 5]. Shipra and Garg [11] studied the effect of heat source/sink on free connective MHD flow past an exponentially accelerated infinite plate with mass diffusion and chemical reaction. Lalrinpuia Tlau and Surender Ontela [6] investigated the generation of entropy in the presence of a heat source/sink in a sloping channel filled with porous medium in magnetohydrodynamic nanofluid flow using Homotopy analysis method. Naveen Dwivedi and Singh [7] studied the fully developed laminar magnetohydrodynamic free convection between two concentric vertical cylinders with Hall currents and heat source/sink, in the presence of the radial magnetic field.

For the composite layers, Sumithra and Manjunatha [12, 13] considered the effect of non-uniform temperature gradients on single and double diffusive magneto-Marangoni convection in a two layer system. They obtained the closed form of solution for Marangoni number. Recently, Sumithra and

Manjunatha [14] studied the effects of heat source/ sink and non uniform temperature gradients on Darcian- Bènard-Magneto-Marangoni convection in composite layer horizontally enclosed by adiabatic boundaries and Sumithra *et al.* [15] have studied the effect of constant heat source / sink on single component Marangoni convection in a composite layer bounded by adiabatic boundaries in presence of uniform & non uniform temperature gradients. They obtained the closed form of solution to composite layer in the presence of constant heat source.

In the present paper an attempt is made to study the effect of non-uniform temperature gradients on Bènard-Surface tension driven convection in a fluid-porous layer in the presence of a constant heat source and vertical magnetic field.

## 2. Formulation of the problem

Consider a horizontal single component, electrically conducting fluid saturated isotropic densely packed porous layer of thickness $d_m$ underlying a single component fluid layer of thickness $d$ with an imposed magnetic field intensity $H_0$ in the vertical z- direction and with heat sources $\Phi_m$ and $\Phi$ respectively. The lower surface of the porous layer rigid and the upper surface of the fluid layer is free with surface tension effects depending on temperature. A Cartesian coordinate system is chosen with the origin at the interface between porous and fluid layers and the z-axis, vertically upwards.
The basic equations for fluid layer

$$\nabla.\overrightarrow{q} = 0 \tag{1}$$

$$\nabla.\overrightarrow{H} = 0 \tag{2}$$

$$\rho_0[\frac{\partial \overrightarrow{q}}{\partial t} + (\overrightarrow{q}.\nabla)\overrightarrow{q}] = -\nabla P + \mu\nabla^2\overrightarrow{q} + \mu_p(\overrightarrow{H}.\nabla)\overrightarrow{H} \tag{3}$$

$$\frac{\partial T}{\partial t} + (\overrightarrow{q}.\nabla)T = \kappa\nabla^2 T + \Phi \tag{4}$$

$$\frac{\partial \overrightarrow{H}}{\partial t} = \nabla \times \overrightarrow{q} \times \overrightarrow{H} + \nu_m\nabla^2\overrightarrow{H} \tag{5}$$

and for porous layer

$$\nabla_m.\overrightarrow{q_m} = 0 \tag{6}$$

$$\nabla_m.\overrightarrow{H} = 0 \tag{7}$$

$$\rho_0[\frac{1}{\varepsilon}\frac{\partial \overrightarrow{q_m}}{\partial t} + \frac{1}{\varepsilon^2}(\overrightarrow{q_m}.\nabla_m)\overrightarrow{q_m}] = -\nabla_m P_m - \frac{\mu}{K}\overrightarrow{q_m} + \mu_p(\overrightarrow{H}.\nabla_m)\overrightarrow{H} \tag{8}$$

$$A\frac{\partial T_m}{\partial t} + (\overrightarrow{q_m}.\nabla_m)T_m = \kappa_m\nabla_m^2 T_m + \Phi_m \tag{9}$$

$$\varepsilon\frac{\partial \overrightarrow{H}}{\partial t} = \nabla_m \times \overrightarrow{q_m} \times \overrightarrow{H} + \nu_{em}\nabla_m^2\overrightarrow{H} \tag{10}$$

where for fluid layer, $\overrightarrow{q}$ is the velocity vector, $\rho_0$ is the fluid density, $t$ is the time, $\mu$ is the fluid viscosity, $P = p + \frac{\mu_p H^2}{2}$ is the total pressure, $\overrightarrow{H}$ is the magnetic field, $T$ is the temperature, constant heat source $\Phi$ , $\kappa$ thermal diffusivity of the fluid, $\nu_m = \frac{1}{\mu_p \sigma}$ is the magnetic viscosity and $\mu_p$ is the magnetic permeability . For porous layer $\varepsilon$ is the porosity , $K$ permeability of the porous medium, $A = \frac{(\rho_0 C_p)_m}{(\rho_0 C_p)_f}$ ratio of heat capacities, $C_p$ specific heat, $\kappa_m$ thermal diffusivity , $\Phi_m$ is constant heat source, $\nu_{em} = \frac{\nu_m}{\varepsilon}$ is the effective magnetic viscosity and the subscripts $'m'$ and $'f'$(in these equations) denotes the quantities in porous layer and fluid layer respectively.

The aim of this paper is to investigate the stability of a quiescent state to infinitesimal perturbations superposed on the basic state. The basic state of the liquid being quiescent is described by

$$\overrightarrow{q} = \overrightarrow{q_b} = 0, P = P_b(z), T = T_b(z), \overrightarrow{H} = H_0(z) \tag{11}$$

$$\overrightarrow{q_m} = \overrightarrow{q_{mb}}, P_m = P_{mb}(z_m), T_m = T_{mb}(z_m), \overrightarrow{H} = H_0(z_m) \tag{12}$$

The basic state temperatures of $T_b(z)$ and $T_{mb}(z_m)$ are obtained as

$$T_b(z) = \frac{-\Phi z(z-d)}{2\kappa} + \frac{(T_u - T_0)h(z)}{d} + T_0 \quad 0 \le z \le d \tag{13}$$

$$T_{mb}(z_m) = \frac{-\Phi_m z_m(z_m + d_m)}{2\kappa_m} + \frac{(T_0 - T_l)h_m(z_m)}{d_m} + T_0 \qquad -d_m \le z_m \le 0 \tag{14}$$

where $T_0 = \dfrac{\kappa d_m T_u + \kappa_m d T_l}{\kappa d_m + \kappa_m d} + \dfrac{d d_m(\Phi_m d_m + \Phi d)}{2(\kappa d_m + \kappa_m d)}$ is the interface temperature and $h(z)$ and $h_m(z_m)$ are the non-dimensional temperature gradients with $\int_0^1 h(z)dz = 1$ and $\int_0^1 h_m(z_m)dz_m = 1$ and subscript $'b'$ denote the basic state.

We superimpose infinitesimal disturbances on the basic state for fluid and porous layer respectively in the form

$$\overrightarrow{q} = \overrightarrow{q_b} + \overrightarrow{q}', P = P_b + P', T = T_b(z) + \theta, \overrightarrow{H} = H_0(z) + \overrightarrow{H'} \tag{15}$$

$$\overrightarrow{q_m} = \overrightarrow{q_{mb}} + \overrightarrow{q_m}', P_m = P_{mb} + P'_m, T_m = T_{mb}(z_m) + \theta_m, \overrightarrow{H} = H_0(z_m) + \overrightarrow{H'} \tag{16}$$

where the prime indicates the perturbations. Introducing (15) and (16) in (1) - (10), operating curl twice and eliminate the pressure term from equations (3) and (8), the resulting equations then nondimensionalized.

The dimensionless equations are then subjected to normal mode analysis as follows

$$\begin{bmatrix} W \\ \theta \\ H \end{bmatrix} = \begin{bmatrix} W(z) \\ \theta(z) \\ H(z) \end{bmatrix} f(x,y)e^{nt} \tag{17}$$

$$\begin{bmatrix} W_m \\ \theta_m \\ H \end{bmatrix} = \begin{bmatrix} W_m(z_m) \\ \theta_m(z_m) \\ H(z_m) \end{bmatrix} f_m(x_m, y_m)e^{n_m t} \tag{18}$$

with $\nabla_2^2 f + a^2 f = 0$ and $\nabla_{2m}^2 f_m + a_m^2 f_m = 0$, where $a$ and $a_m$ are the nondimensional horizontal wave numbers, $n$ and $n_m$ are the frequencies, $W(z)$ and $W_m(z_m)$ are the dimensionless vertical velocities in fluid and porous layer respectively, $\theta(z)$ and $\theta_m(z_m)$ are the temperature distributions in fluid and porous layers respectively and obtain an eigenvalue problem consisting of the following ordinary differential equations
in $0 \leq z \leq 1$

$$(D^2 - a^2 + \frac{n}{Pr})(D^2 - a^2)W(z) = -Q\tau_{fm}D(D^2 - a^2)H(z) \tag{19}$$

$$(D^2 - a^2 + n)\theta(z) + [h(z) + R_I^*(2z - 1)]W(z) = 0 \tag{20}$$

$$(\tau_{fm}(D^2 - a^2) + n)H(z) + DW(z) = 0 \tag{21}$$

in $-1 \leq z_m \leq 0$

$$(1 - \frac{\beta^2 n_m}{P_{rm}})(D_m^2 - a_m^2)W_m(z_m) = Q_m\tau_{mm}\beta^2 D_m(D_m^2 - a_m^2)H(z_m) \tag{22}$$

$$(D_m^2 - a_m^2 + An_m)\theta_m(z_m) + [h_m(z_m) + R_{Im}^*(2z_m + 1)]W_m(z_m) = 0 \tag{23}$$

$$(\tau_{mm}(D_m^2 - a_m^2) + n_m\varepsilon)H(z_m) + D_m W_m(z_m) = 0 \tag{24}$$

Here for fluid layer, $Pr = \frac{\nu}{\kappa}$ is the prandtl number, $Q = \frac{\mu_p H_0^2 d^2}{\mu\kappa\tau_{fm}}$ is the Chandrasekhar number, $\tau_{fm} = \frac{\nu_{mv}}{\kappa}$ is the diffusivity ratio. For porous layer, $Pr_m = \frac{\varepsilon\nu_m}{\kappa_m}$ is the prandtl number, $Q_m = \frac{\mu_p H_0^2 d_m^2}{\mu\kappa_m\tau_{mm}} = Q\varepsilon\widehat{d}^2$ is the Chandrasekhar number, $\beta = \sqrt{\frac{K}{d_m^2}}$ is the porous parameter, $\tau_{mm} = \frac{\nu_{em}}{\kappa_m}$ is the diffusivity ratio. $R_I^* = \frac{R_I}{2(T_0 - T_u)}$ , $R_{Im}^* = \frac{R_{Im}}{2(T_l - T_0)}$, here $R_I$ is the internal Rayleigh number for fluid layer and $R_{Im}$ is the internal Rayleigh number for porous layer.
Substituting the equation (21) in (19) and (24) in (22) to eliminate the magnetic field and assume that the present problem is satisfies the principle of exchange of stability, so putting $n = n_m = 0$. We get
in $0 \leq z \leq 1$

$$(D^2 - a^2)^2 W(z) = QD^2 W(z) \tag{25}$$

$$(D^2 - a^2)\theta(z) + [h(z) + R_I^*(2z - 1)]W(z) = 0 \tag{26}$$

in $-1 \leq z_m \leq 0$

$$(D_m^2 - a_m^2)W_m(z_m) = -Q_m\beta^2 D_m^2 W_m(z_m) \tag{27}$$

$$(D_m^2 - a_m^2)\theta_m(z_m) + [h_m(z_m) + R_{Im}^*(2z_m + 1)]W_m(z_m) = 0 \tag{28}$$

## 3. BOUNDARY CONDITIONS

The boundary conditions are nondimensionalized and then subjected to normal mode expansion and are

$$\begin{aligned} D^2W(1) + Ma^2\theta(1) = 0, \\ W(1) = 0, W_m(-1) = 0, \widehat{T}W(0) = W_m(0), \\ \widehat{T}\widehat{d}DW(0) = D_mW_m(0), \\ \widehat{T}\widehat{d}^3\beta^2(D^3W(0) - 3a^2DW(0)) = -D_mW_m(0), \\ \theta(1) = 0, \theta(0) = \widehat{T}\theta_m(0), \\ D\theta(0) = D_m\theta_m(0), D_m\theta_m(-1) = 0 \end{aligned} \tag{29}$$

where
$\widehat{T} = \dfrac{T_l - T_0}{T_0 - T_u}$ is the thermal ratio, $M = -\dfrac{\partial\sigma_t}{\partial T}\dfrac{(T_0 - T_u)d}{\mu\kappa}$ is the thermal Marangoni number and $\widehat{d} = \dfrac{d_m}{d}$ is the depth ratio.

## 4. METHOD OF SOLUTION

The solutions $W(z)$ and $W_m(z_m)$ are obtained by solving (25) and (27) using the velocity boundary conditions (29)

$$W(z) = A_1[\cosh\delta z + a_1\sinh\delta z + a_2\cosh\zeta z + a_3\sinh\zeta z] \tag{30}$$

$$W_m(z_m) = A_1[a_4\cosh\delta_m z_m + a_5\sinh\delta_m z_m] \tag{31}$$

where

$$\delta = \frac{\sqrt{Q} - \sqrt{Q + 4a^2}}{2}, \zeta = \frac{\sqrt{Q} + \sqrt{Q + 4a^2}}{2}, \delta_m = \sqrt{\frac{a_m^2}{1 + Q_m\beta^2}}$$

$$a_1 = -\frac{\Delta_2 a_3}{\Delta_1}, a_2 = \frac{\Delta_5\Delta_7 - \Delta_8\Delta_4}{\Delta_3\Delta_7 - \Delta_6\Delta_4}, a_3 = \frac{\Delta_5\Delta_6 - \Delta_8\Delta_3}{\Delta_4\Delta_6 - \Delta_7\Delta_3},$$

$$a_4 = \widehat{T}(1 + a_2), a_5 = \frac{1}{\delta_m}(\widehat{T}\widehat{d}a_1\delta + a_3\zeta)$$

$$\Delta_1 = \widehat{d}^2\beta^2(\delta^3 - 3a^2\delta) + \delta, \Delta_2 = \widehat{d}^2\beta^2(\zeta^3 - 3a^2\zeta) + \zeta,$$

$$\Delta_3 = \widehat{T}\cosh\delta_m, \Delta_4 = -\frac{\widehat{d}\widehat{T}\sinh\delta_m}{\delta_m}(\zeta - \frac{\Delta_2\delta}{\Delta_1}),$$

$$\Delta_5 = -\Delta_3, \Delta_6 = \cosh\zeta, \Delta_7 = \sinh\zeta - (\tfrac{\Delta_2}{\Delta_1})\sinh\delta, \Delta_8 = -\cosh\delta$$

### 4.1. *Linear temperature profile.*

Consider the linear profile of the form

$$h(z) = 1 \quad and \quad h_m(z_m) = 1 \tag{32}$$

Substituting equation (32) into (26) and (28), the temperature distributions $\theta(z)$ and $\theta_m(z_m)$ are obtained using the temperature boundary conditions, as follows

$$\theta(z) = A_1[c_1 \cosh az + c_2 \sinh az + g_1(z)] \tag{33}$$

$$\theta_m(z_m) = A_1[c_3 \cosh a_m z_m + c_4 \sinh a_m z_m + g_{m1}(z_m)] \tag{34}$$

where

$g_1(z) = A_1[\delta_1 - \delta_2 + \delta_3 - \delta_4], g_{m1}(z_m) = A_1[\delta_5 - \delta_6]$

$\delta_1 = \dfrac{(E_2 z + E_1)}{(\delta^2 - a^2)}(\cosh \delta z + a_1 \sinh \delta z)$

$\delta_2 = \dfrac{2\delta E_2}{(\delta^2 - a^2)^2}(a_1 \cosh \delta z + \sinh \delta z)$

$\delta_3 = \dfrac{(E_2 z + E_1)}{(\zeta^2 - a^2)}(a_2 \cosh \zeta z + a_3 \sinh \zeta z)$

$\delta_4 = \dfrac{2\zeta E_2}{(\zeta^2 - a^2)^2}(a_3 \cosh \zeta z + a_2 \sinh \zeta z)$

$\delta_5 = \dfrac{(E_{1m} + E_{2m} z_m)}{(\delta_m^2 - a_m^2)}(a_4 \cosh \delta_m z_m + a_5 \sinh \delta_m z_m)$

$\delta_6 = \dfrac{2E_{2m}\delta_m}{(\delta_m^2 - a_m^2)^2}(a_5 \cosh \delta_m z_m + a_4 \sinh \delta_m z_m)$

$E_1 = R_I^* - 1, E_2 = -2R_I^*, E_{1m} = -(R_{Im}^* + 1), E_{2m} = -2R_{Im}^*$

$c_1 = c_3 \widehat{T} + \Delta_{10} - \Delta_{11}, c_2 = \frac{1}{a}(c_4 a_m + \Delta_{12} - \Delta_{13}),$

$c_3 = \dfrac{\Delta_{19}\Delta_{14} - \Delta_{16}\Delta_{18}}{\Delta_{15}\Delta_{18} + \Delta_{17}\Delta_{14}}, c_4 = \dfrac{\Delta_{16}\Delta_{17} + \Delta_{19}\Delta_{15}}{\Delta_{14}\Delta_{17} + \Delta_{18}\Delta_{15}},$

$\Delta_9 = -[\delta_7 - \delta_8 + \delta_9 - \delta_{10}],$

$\delta_7 = \dfrac{(E_2 + E_1)}{(\delta^2 - a^2)}(\cosh \delta + a_1 \sinh \delta),$

$\delta_8 = [\dfrac{2\delta E_2}{(\delta^2 - a^2)^2}](a_1 \cosh \delta + \sinh \delta),$

$\delta_9 = \dfrac{(E_2 + E_1)}{(\zeta^2 - a^2)}(a_2 \cosh \zeta + a_3 \sinh \zeta),$

$\delta_{10} = [\dfrac{2\zeta E_2}{(\zeta^2 - a^2)^2}](a_3 \cosh \zeta + a_2 \sinh \zeta),$

$\Delta_{10} = \widehat{T}[\dfrac{E_{1m} a_4}{(\delta_m^2 - a_m^2)} - \dfrac{2E_{2m}\delta_m a_5}{(\delta_m^2 - a_m^2)^2}],$

$\Delta_{11} = \dfrac{E_1}{(\delta^2 - a^2)} - \dfrac{2\delta a_1 E_2}{(\delta^2 - a^2)^2} + \dfrac{a_2 E_1}{(\zeta^2 - a^2)} - \dfrac{2\zeta a_3 E_2}{(\zeta^2 - a^2)^2},$

$\Delta_{12} = [\dfrac{E_{2m}}{(\delta_m^2 - a_m^2)} - \dfrac{2\delta_m^2 E_{2m}}{(\delta_m^2 - a_m^2)^2}]a_4 + \dfrac{a_5 E_{1m}}{(\delta_m^2 - a_m^2)}$

$\Delta_{13} = \dfrac{E_1 \delta a_1 + E_2}{(\delta^2 - a^2)} - \dfrac{2E_2\delta^2}{(\delta^2 - a^2)^2} + \dfrac{E_1 \zeta a_3 + E_2 a_2}{(\zeta^2 - a^2)} - \dfrac{2a_2 E_2 \zeta^2}{(\zeta^2 - a^2)^2},$

$\Delta_{14} = a_m \cosh a_m, \Delta_{15} = a_m \sinh a_m,$

$\Delta_{16} = -[\dfrac{E_{2m}}{(\delta_m^2 - a_m^2)} - \dfrac{2\delta_m^2 E_{2m}}{(\delta_m^2 - a_m^2)^2}](a_4 \cosh \delta_m - a_5 \sinh \delta_m) - \Delta_{160}$

$\Delta_{160} = \dfrac{\delta_m(E_{1m} - E_{2m})}{(\delta_m^2 - a_m^2)}(a_5 \cosh \delta_m - a_4 \sinh \delta_m),$

$\Delta_{17} = \widehat{T}\cosh a, \Delta_{18} = \dfrac{a_m \sinh a}{a},$
$\Delta_{19} = \Delta_9 - (\Delta_{10} - \Delta_{11})\cosh a - \dfrac{1}{a}(\Delta_{12} - \Delta_{13})\sinh a$
From the boundary condition (29), we have

$$M = \frac{-D^2W(1)}{a^2\theta(1)}$$

The thermal Marangoni number for the linear temperature profile is as follows

$$M_1 = -\frac{[\delta^2(\cosh\delta + a_1\sinh\delta) + \zeta^2(a_2\cosh\zeta + a_3\sinh\zeta)]}{a^2(c_1\cosh a + c_2\sinh a + \Lambda_1 + \Lambda_2)} \tag{35}$$

where
$\Lambda_1 = \dfrac{(E_2 + E_1)}{(\delta^2 - a^2)}(\cosh\delta + a_1\sinh\delta) - \dfrac{2\delta E_2}{(\delta^2 - a^2)^2}(a_1\cosh\delta + \sinh\delta)$
$\Lambda_2 = \dfrac{(E_2 + E_1)}{(\zeta^2 - a^2)}(a_2\cosh\zeta + a_3\sinh\zeta) - \dfrac{2\zeta E_2}{(\zeta^2 - a^2)^2}(a_3\cosh\zeta + a_2\sinh\zeta)$

### 4.2. ***Parabolic temperature profile.***

The parabolic temperature profile of the form

$$h(z) = 2z \quad and \quad h_m(z_m) = 2z_m \tag{36}$$

Substituting (36) into (26) and (28), the temperature distributions $\theta(z)$ and $\theta_m(z_m)$ are obtained using the temperature boundary conditions is as follows

$$\theta(z) = A_1[c_5\cosh az + c_6\sinh az + g_2(z)] \tag{37}$$

$$\theta_m(z_m) = A_1[c_7\cosh a_m z_m + c_8\sinh a_m z_m + g_{m2}(z_m)] \tag{38}$$

where
$g_2(z) = A_1[\delta_{11} - \delta_{12} + \delta_{13} - \delta_{14}], g_{m2}(z_m) = A_1[\delta_{15} - \delta_{16}]$
$\delta_{11} = \dfrac{(E_4 z + E_3)}{(\delta^2 - a^2)}(\cosh\delta z + a_1\sinh\delta z)$
$\delta_{12} = \dfrac{2\delta E_4}{(\delta^2 - a^2)^2}(a_1\cosh\delta z + \sinh\delta z)$
$\delta_{13} = \dfrac{(E_4 z + E_3)}{(\zeta^2 - a^2)}(a_2\cosh\zeta z + a_3\sinh\zeta z)$
$\delta_{14} = \dfrac{2\zeta E_4}{(\zeta^2 - a^2)^2}(a_3\cosh\zeta z + a_2\sinh\zeta z)$
$\delta_{15} = \dfrac{(E_{3m} + E_{4m}z_m)}{(\delta_m^2 - a_m^2)}(a_4\cosh\delta_m z_m + a_5\sinh\delta_m z_m)$
$\delta_{16} = \dfrac{2E_{4m}\delta_m}{(\delta_m^2 - a_m^2)^2}(a_5\cosh\delta_m z_m + a_4\sinh\delta_m z_m),$
$E_3 = R_I^*, E_4 = -2(R_I^* + 1), E_{3m} = -R_{Im}^*, E_{4m} = -2(R_{Im}^* + 1)$
$c_5 = c_7\widehat{T} + \Delta_{21} - \Delta_{22}, c_6 = \frac{1}{a}(c_8 a_m + \Delta_{23} - \Delta_{24}),$
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$c_7 = \dfrac{\Delta_{30}\Delta_{25} - \Delta_{27}\Delta_{29}}{\Delta_{28}\Delta_{25} + \Delta_{26}\Delta_{29}}, c_8 = \dfrac{\Delta_{30}\Delta_{26} + \Delta_{27}\Delta_{28}}{\Delta_{29}\Delta_{26} + \Delta_{25}\Delta_{28}},$
$\Delta_{20} = -[\delta_{17} - \delta_{18} + \delta_{19} - \delta_{20}],$
$\delta_{17} = \dfrac{(E_4 + E_3)}{(\delta^2 - a^2)}(\cosh\delta + a_1\sinh\delta),$
$\delta_{18} = [\dfrac{2\delta E_4}{(\delta^2 - a^2)^2}](a_1\cosh\delta + \sinh\delta),$
$\delta_{19} = \dfrac{(E_4 + E_3)}{(\zeta^2 - a^2)}(a_2\cosh\zeta + a_3\sinh\zeta),$
$\delta_{20} = [\dfrac{2\zeta E_4}{(\zeta^2 - a^2)^2}](a_3\cosh\zeta + a_2\sinh\zeta),$
$\Delta_{21} = \widehat{T}[\dfrac{E_{3m}a_4}{(\delta_m^2 - a_m^2)} - \dfrac{2E_{4m}\delta_m a_5}{(\delta_m^2 - a_m^2)^2}],$
$\Delta_{22} = \dfrac{E_3}{(\delta^2 - a^2)} - \dfrac{2\delta a_1 E_4}{(\delta^2 - a^2)^2} + \dfrac{a_2 E_3}{(\zeta^2 - a^2)} - \dfrac{2\zeta a_3 E_4}{(\zeta^2 - a^2)^2},$
$\Delta_{23} = [\dfrac{E_{4m}}{(\delta_m^2 - a_m^2)} - \dfrac{2\delta_m^2 E_{4m}}{(\delta_m^2 - a_m^2)^2}]a_4 + \dfrac{a_5 E_{3m}}{(\delta_m^2 - a_m^2)}$
$\Delta_{24} = \dfrac{E_3\delta a_1 + E_4}{(\delta^2 - a^2)} - \dfrac{2E_4\delta^2}{(\delta^2 - a^2)^2} + \dfrac{E_3\zeta a_3 + E_4 a_2}{(\zeta^2 - a^2)} - \dfrac{2a_2 E_4\zeta^2}{(\zeta^2 - a^2)^2},$
$\Delta_{25} = a_m\cosh a_m, \Delta_{26} = a_m\sinh a_m,$
$\Delta_{27} = -[\dfrac{E_{4m}}{(\delta_m^2 - a_m^2)} - \dfrac{2\delta_m^2 E_{4m}}{(\delta_m^2 - a_m^2)^2}](a_4\cosh\delta_m - a_5\sinh\delta_m) - \Delta_{270}$
$\Delta_{270} = \dfrac{\delta_m(E_{3m} - E_{4m})}{(\delta_m^2 - a_m^2)}(a_5\cosh\delta_m - a_4\sinh\delta_m),$
$\Delta_{28} = \widehat{T}\cosh a, \Delta_{29} = \frac{1}{a}a_m\sinh a,$
$\Delta_{30} = \Delta_{20} - (\Delta_{21} - \Delta_{22})\cosh a - \frac{1}{a}(\Delta_{23} - \Delta_{24})\sinh a$
From the boundary condition (29), the thermal Marangoni number for parabolic temperature profile is as follows

$$M_2 = -\frac{[\delta^2(\cosh\delta + a_1\sinh\delta) + \zeta^2(a_2\cosh\zeta + a_3\sinh\zeta)]}{a^2(c_5\cosh a + c_6\sinh a + \Lambda_3 + \Lambda_4)} \tag{39}$$

where
$\Lambda_3 = \dfrac{(E_4 + E_3)}{(\delta^2 - a^2)}(\cosh\delta + a_1\sinh\delta) - \dfrac{2\delta E_4}{(\delta^2 - a^2)^2}(a_1\cosh\delta + \sinh\delta)$
$\Lambda_4 = \dfrac{(E_4 + E_3)}{(\zeta^2 - a^2)}(a_2\cosh\zeta + a_3\sinh\zeta) - \dfrac{2\zeta E_4}{(\zeta^2 - a^2)^2}(a_3\cosh\zeta + a_2\sinh\zeta)$

### 4.3. ***Inverted Parabolic temperature profile.***

Consider the inverted parabolic profile of the form

$$h(z) = 2(1 - z) \quad and \quad h_m(z_m) = 2(1 - z_m) \tag{40}$$

Substituting (40) into (26) and (28), the temperature distributions $\theta(z)$ and $\theta_m(z_m)$ are obtained using the temperature boundary conditions, as follows

$$\theta(z) = A_1[c_9 \cosh az + c_{10} \sinh az + g_3(z)] \tag{41}$$

$$\theta_m(z_m) = A_1[c_{11} \cosh a_m z_m + c_{12} \sinh a_m z_m + g_{m3}(z_m)] \tag{42}$$

where

$g_3(z) = A_1[\delta_{21} - \delta_{22} + \delta_{23} - \delta_{24}], g_{m3}(z_m) = A_1[\delta_{25} - \delta_{26}]$

$\delta_{21} = \dfrac{(E_6 z + E_5)}{(\delta^2 - a^2)}(\cosh \delta z + a_1 \sinh \delta z)$

$\delta_{22} = \dfrac{2\delta E_6}{(\delta^2 - a^2)^2}(a_1 \cosh \delta z + \sinh \delta z)$

$\delta_{23} = \dfrac{(E_6 z + E_5)}{(\zeta^2 - a^2)}(a_2 \cosh \zeta z + a_3 \sinh \zeta z)$

$\delta_{24} = \dfrac{2\zeta E_6}{(\zeta^2 - a^2)^2}(a_3 \cosh \zeta z + a_2 \sinh \zeta z)$

$\delta_{25} = \dfrac{(E_{5m} + E_{6m} z_m)}{(\delta_m^2 - a_m^2)}(a_4 \cosh \delta_m z_m + a_5 \sinh \delta_m z_m)$

$\delta_{26} = \dfrac{2E_{6m}\delta_m}{(\delta_m^2 - a_m^2)^2}(a_5 \cosh \delta_m z_m + a_4 \sinh \delta_m z_m),$

$E_5 = R_I^* - 2, E_6 = 2(1 - R_I^*), E_{5m} = -2 - R_{Im}^*, E_{6m} = 2(1 - R_{Im}^*)$

$c_9 = c_{11}\widehat{T} + \Delta_{32} - \Delta_{33}, c_{10} = \frac{1}{a}(c_{12}a_m + \Delta_{34} - \Delta_{35}),$

$c_{11} = \dfrac{\Delta_{36}\Delta_{41} - \Delta_{38}\Delta_{40}}{\Delta_{39}\Delta_{36} + \Delta_{37}\Delta_{40}}, c_{12} = \dfrac{\Delta_{41}\Delta_{37} + \Delta_{39}\Delta_{38}}{\Delta_{40}\Delta_{37} + \Delta_{36}\Delta_{39}},$

$\Delta_{31} = -[\delta_{27} - \delta_{28} + \delta_{29} - \delta_{30}],$

$\delta_{27} = \dfrac{(E_6 + E_5)}{(\delta^2 - a^2)}(\cosh \delta + a_1 \sinh \delta),$

$\delta_{28} = [\dfrac{2\delta E_6}{(\delta^2 - a^2)^2}](a_1 \cosh \delta + \sinh \delta),$

$\delta_{29} = \dfrac{(E_6 + E_5)}{(\zeta^2 - a^2)}(a_2 \cosh \zeta + a_3 \sinh \zeta),$

$\delta_{30} = [\dfrac{2\zeta E_6}{(\zeta^2 - a^2)^2}](a_3 \cosh \zeta + a_2 \sinh \zeta),$

$\Delta_{32} = \widehat{T}[\dfrac{E_{5m}a_4}{(\delta_m^2 - a_m^2)} - \dfrac{2E_{6m}\delta_m a_5}{(\delta_m^2 - a_m^2)^2}],$

$\Delta_{33} = \dfrac{E_5}{(\delta^2 - a^2)} - \dfrac{2\delta a_1 E_6}{(\delta^2 - a^2)^2} + \dfrac{a_2 E_5}{(\zeta^2 - a^2)} - \dfrac{2\zeta a_3 E_6}{(\zeta^2 - a^2)^2},$

$\Delta_{34} = [\dfrac{E_{6m}}{(\delta_m^2 - a_m^2)} - \dfrac{2\delta_m^2 E_{6m}}{(\delta_m^2 - a_m^2)^2}]a_4 + \dfrac{a_5 E_{5m}}{(\delta_m^2 - a_m^2)}$

$\Delta_{35} = \dfrac{E_5\delta a_1 + E_6}{(\delta^2 - a^2)} - \dfrac{2E_6\delta^2}{(\delta^2 - a^2)^2} + \dfrac{E_5\zeta a_3 + E_6 a_2}{(\zeta^2 - a^2)} - \dfrac{2a_2 E_6 \zeta^2}{(\zeta^2 - a^2)^2},$

$\Delta_{36} = a_m \cosh a_m, \Delta_{37} = a_m \sinh a_m,$

$\Delta_{38} = -[\dfrac{E_{6m}}{(\delta_m^2 - a_m^2)} - \dfrac{2\delta_m^2 E_{6m}}{(\delta_m^2 - a_m^2)^2}](a_4 \cosh \delta_m - a_5 \sinh \delta_m) - \Delta_{380}$

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$\Delta_{380} = \dfrac{\delta_m(E_{5m} - E_{6m})}{(\delta_m^2 - a_m^2)}(a_5 \cosh \delta_m - a_4 \sinh \delta_m),$
$\Delta_{39} = \widehat{T} \cosh a, \Delta_{40} = \frac{1}{a} a_m \sinh a,$
$\Delta_{41} = \Delta_{31} - (\Delta_{32} - \Delta_{33}) \cosh a - \frac{1}{a}(\Delta_{34} - \Delta_{35}) \sinh a$
From the boundary condition (29), the thermal Marangoni number for inverted parabolic temperature profile is as follows

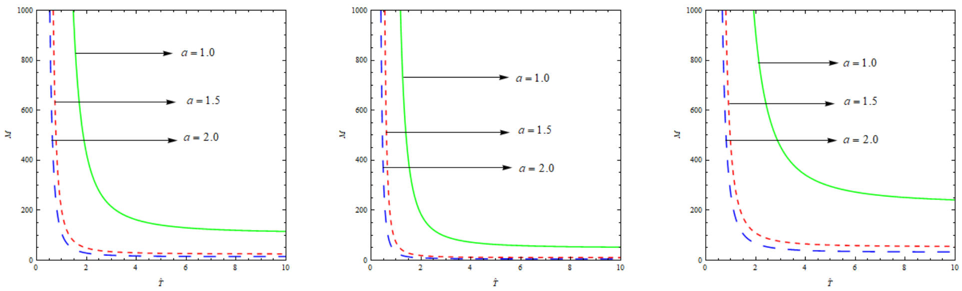
$$M_3 = -\frac{[\delta^2(\cosh \delta + a_1 \sinh \delta) + \zeta^2(a_2 \cosh \zeta + a_3 \sinh \zeta)]}{a^2(c_9 \cosh a + c_{10} \sinh a + \Lambda_5 + \Lambda_6)} \tag{43}$$

where
$\Lambda_5 = \dfrac{(E_6 + E_5)}{(\delta^2 - a^2)}(\cosh \delta + a_1 \sinh \delta) - \dfrac{2\delta E_6}{(\delta^2 - a^2)^2}(a_1 \cosh \delta + \sinh \delta)$
$\Lambda_6 = \dfrac{(E_6 + E_5)}{(\zeta^2 - a^2)}(a_2 \cosh \zeta + a_3 \sinh \zeta) - \dfrac{2\zeta E_6}{(\zeta^2 - a^2)^2}(a_3 \cosh \zeta + a_2 \sinh \zeta).$

## 5. Results and Discussion

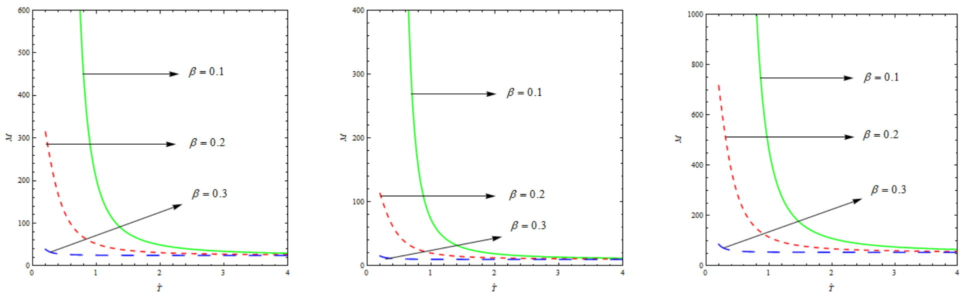
The Eigen value thermal Marangoni numbers $M$ for linear, parabolic and inverted parabolic temperature profiles are obtained as an expression of the horizontal wavenumbers $a$ and $a_m$, the Chandrasekhar number $Q$, the porous parameter $\beta$, the thermal ratio $\widehat{T}$, the internal Rayleigh numbers $R_I$ and $R_{Im}$ for the fluid and porous layers respectively and the depth ratio $\widehat{d}$. From the graphs it is clear that, for lower values of thermal ratio, the thermal Marangoni numbers decreases and then remain constant for larger values of thermal ratios. That is when the value of $\widehat{T} < 3$ the curves are vertical and the thermal Marangoni number decreases almost vertically and then the curves become horizontal, in otherwords, the thermal Marangoni number remains constant for values of $\widehat{T} > 3$. The effects of the parameters $a, \beta, Q, R_I$ and $\widehat{d}$ on the thermal Marangoni numbers $M$ are shown in the following figures for linear, Parabolic and inverted parabolic temperature profiles for fixed values of $a = 1.0, \beta = 0.1, Q = 5, R_I = -1, R_{Im} = 1$ and $\widehat{d} = 2.5$ as a function of thermal ratio $\widehat{T}$.

The effects of the horizontal wavenumber $a$ on the thermal Marangoni number is exhibited in figures 1a,1b and 1c for linear, parabolic and inverted parabolic temperature profiles respectively and they are for $a = 1.0, 1.5$ and $2.0$. The effects of $a$ is similar for all the three profiles and for a fixed value of thermal ratio, the increase in the value of the horizontal wavenumber for the fluid layer , there is a decrease in the value of the thermal Marangoni number. That is the Bènard-Surface tension driven convection is augmented favoring the situations demanding convection in the presence of magnetic field. Hence the increase in the value of $a$ destabilizes the system which is conducive for the situations which require convection namely heat transfer problems.
The effects of the porous parameter $\beta$ on the thermal Marangoni number is




(a) (b) (c)

FIGURE 1. Effects of horizontal wave number $a$




(a) (b) (c)

FIGURE 2. Effects of porous parameter $\beta$

displayed in the figures 2a,2b and 2c respectively for linear, parabolic and inverted parabolic temperature profiles for $\beta = 0.1, 0.2$ and $0.3$. The effects of $\beta$ is parallel for all the three profiles and for a fixed value of thermal ratio, the increase in the value of the porous parameter $\beta$ , decreases the value of the thermal Marangoni number. That is the Bènard-Surface tension driven convection in the presence of magnetic field is preponed. That is, more window for the fluid destabilizes the system which is physically reasonable. Also the converging curves indicate that the effect of the porous parameter is drastic for the smaller values of thermal ratios, hence this parameter plays a prominent role in the composite systems where $T_l - T_0 < T_0 - T_u$.
The effects of the Chandrasekhar number $Q$ on the thermal Marangoni numbers is depicted in figures 3a,3b and 3c for linear, parabolic and inverted parabolic temperature profiles respectively and they are for $Q = 5, 25$ and $50$. The effects of $Q$ is same for all the three profiles and for a fixed value of thermal ratio, the increase in the value of Chandrasekhar number $Q$, increases the value of the thermal Marangoni number. That is the Bènard-Surface tension driven convection is postponed when the strength of the magnetic field is increased. Hence the presence of magnetic field stabilizes the system.

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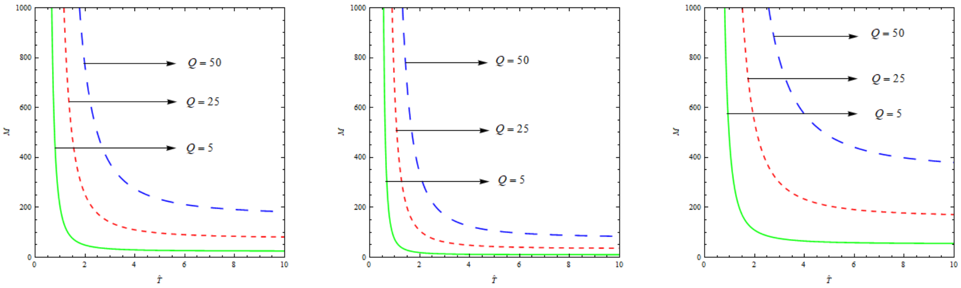
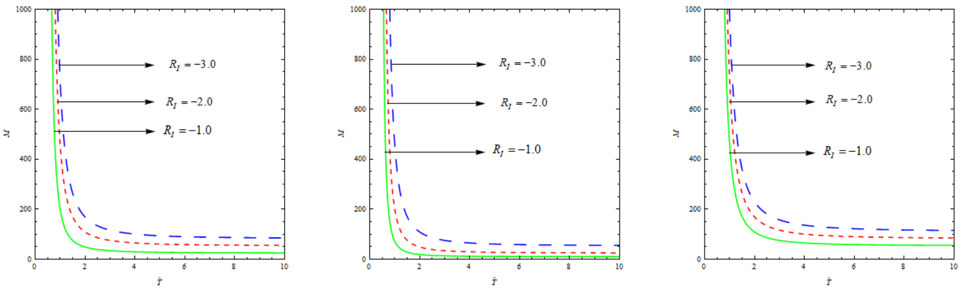



(a) (b) (c)

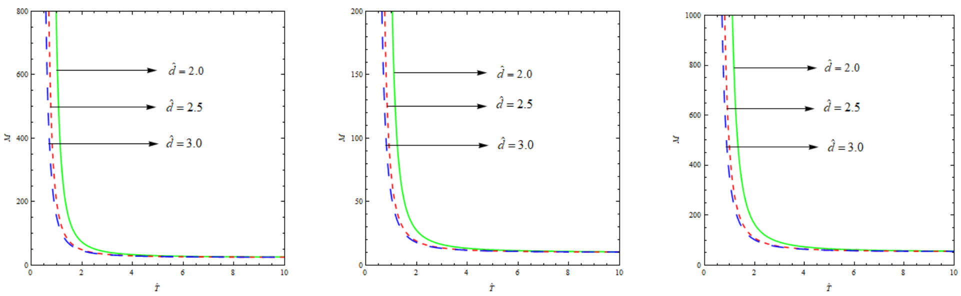
FIGURE 3. Effects of Chandrasekhar number $Q$




(a) (b) (c)

FIGURE 4. Effects of internal Rayleigh number $R_I$

The effect of internal Rayleigh number $R_I$ on the Marangoni number is similar for all the three temperature profiles depicted figures 4a, 4b and 4c for $R_I = -1, -2$ and $-3$. Decreasing the values of $R_I$, the Marangoni number increases, which is physically reasonable as the absorption of heat stabilizes the system , hence the Bènard-Surface tension driven convection in the presence of magnetic field can be deferred by decreasing the values of $R_I$.




(a) (b) (c)

FIGURE 5. Effects of depth ratio $\widehat{d}$

The effects of the depth ratio $\widehat{d}$ on the thermal Marangoni numbers is presented in figures 5a,5b and 5c for linear, parabolic and inverted parabolic temperature profiles respectively and they are for $\widehat{d} = 2.0, 2.5$ and $3.0$. The effects of depth ratio $\widehat{d}$ is identical for all the three profiles and for a fixed value of thermal ratio, the increase in the value of depth ratio $\widehat{d}$, decreases the value of the thermal Marangoni number. That is the Bènard-Surface tension driven convection in the presence of magnetic field is preponed. Hence the system is destabilized. The converging curves indicate that the effect of the depth ratio if prominent for smaller values of thermal ratios.

## 6. Conclusion

Following conclusions are drawn from this study

(i) The effects of the physical parameters is analogous for uniform and non uniform temperature gradients considered in the study.
(ii) The inverted parabolic temperature gradient is the highly stable of all the three gradients.
(iii) By reducing the values of horizontal wavenumber $a$, the porous parameter $\beta$, the internal Rayleigh numbers $R_I$ and the depth ratio $\widehat{d}$ , one can prepone the Bènard- Surface tension driven convection in the presence of magnetic field.
(iv) By increasing the value of Chandrasekhar number $Q$ Bènard-Surface tension driven convection in the presence of magnetic field can be postponed.
(v) There is no effect of internal Rayleigh number $R_{Im}$ on the Bènard-Surface tension driven convection in the presence of vertical magnetic field.
(vi) The parameters the porous parameter $\beta$ and the depth ratio $\widehat{d}$ play an vital role for the composite layer with $T_l - T_0 < T_0 - T_u$.

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