EDGE DOMINATION IN SOME IMAGE GRAPHS

Anjaneyulu Mekala [1]

Department of Mathematics, Guru Nanak Institutions Technical Campus (Autonomous), Hyderabad, Telangana, India.

U. Vijaya Chandara Kumar [1]

School of Applied Sciences, Department of Mathematics, REVA University, Bengaluru, Karnataka, India.

R Murali[2]

Department of Mathematics, Dr. Ambedkar Institute of Technology, Bengaluru

Abstract

Let $G = (V, E)$ be a simple connected and undirected graph. The shadow distance graph of G, denoted by $D_{sd}(G, D_s)$ is constructed from G with the following conditions:

i) consider two copies of G say G itself and $G^{'}$

ii) if $u \in V(G)$ (first copy) then we denote the corresponding vertex as $u^{'} \in V(G^{'})$ (second copy)

iii) the vertex set of $D_{sd}(G, D_s)$ is $V(G) \cup V(G^{'})$

iv) the edge set of $D_{sd}(G, D_s)$ is $E(G) \cup E(G^{'}) \cup E_{ds}$ where E_{ds} is the set of all edges between two distinct vertices $u \in V(G)$ and $v^{'} \in V(G^{'})$ that satisfy the condition $d(u, v) \in D_s$ in G.

The image graph of a connected graph G, denoted by $I_{mg}(G)$ is the graph obtained by joining the vertices of the original graph G to the corresponding vertices of a copy of G. In this paper, we determine the edge domination number of shadow distance graph of the image path graph, the image cycle graph and the image sunlet graph with specified distance set.

Key Words: Dominating set, Minimal edge dominating set, Minimal Domination number, Shadow distance graph, Image graph.

AMS Subject Classification: 05C69.

1. INTRODUCTION

By a graph $G = (V, E)$ we mean a finite undirected graph without loops and multiple edges. A subset D of V is called a dominating set of G if every vertex not in D is adjacent to some vertex in D. The domination number of G denoted by $\gamma(G)$ is the minimal cardinality taken over all dominating sets of G. A vertex v in a graph G dominates the vertices in its closed neighbourhood $N[v]$, that is, v is said to dominating itself and each of its neighbours.

A subset F of E is called an edge dominating set if each edge in E is either in F or is adjacent to an edge in F. An edge dominating set F is called minimal if no proper subset of F is an edge dominating set. The edge domination number of G denoted by $\gamma^{'}(G)$ is the minimum cardinality taken over all edge dominating sets of G.

The concept of edge domination in graphs was introduced by Mitchell and Hedetniemi [5]. Arumugam and Velammal[6] have characterized connected graphs for which $\gamma^{'}(G) = \lfloor \frac{V}{2} \rfloor$ and in [7], Armugam and Jerry have studied edge domination and fractional edge domination in graphs. In [5], Vaidya and Pandit have discussed edge domination in some path and cycle related graphs and have determined the edge domination number for shadow graphs, middle graph, and total graphs of paths and cycles.

The open neighbourhood of an edge $e \in E$ denoted by $N(e)$ is the set of all edges adjacent to e in G. If $e = (u, v)$ is an edge in G, the degree of e denoted by $deg(e)$ is defined as $deg(e) = deg(u)+deg(v)-2$. The maximum degree of an edge in G is denoted by $\triangle^{'}(G)$. The shadow graph of G, denoted by $D_2(G)$ is the graph constructed from G by taking two copies of G namely G itself and $G^{'}$ and by joining each vertex u in G to the neighbors of the corresponding vertex $u^{'}$ in $G^{'}$. Let D be the set of all distances between distinct pairs of vertices in G and let D_s (called the distance set) be a subset of D. The distance graph of G denoted by $D(G, D_s)$ is the graph having the same vertex set as that of G and two vertices u and v are adjacent in $D(G, D_s)$ whenever $d(u, v) \in D_s$.

The shadow graph of G, denoted by $D_2(G)$ is the graph constructed from G by taking two copies of G, namely G itself and $G^{'}$ and by joining each vertex u in G to the neighbors of the corresponding vertex $u^{'}$ in $G^{'}$.

Let D be the set of all distances between distinct pairs of vertices in G and let D_s (called the distance set) be a subset of D. The distance graph of G denoted by $D(G, D_s)$ is the graph having the same vertex set as that of G and two vertices u and v are adjacent in $D(G, D_s)$ whenever $d(u, v) \in D_s$.

The shadow distance graph [2] of G, denoted by $D_{sd}(G, D_s)$ is constructed from G with the following conditions:

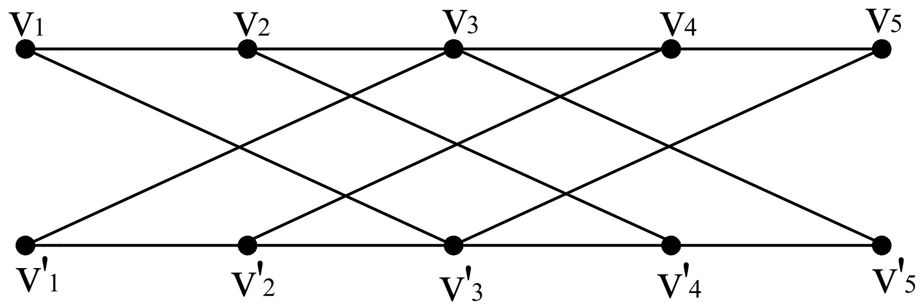

FIGURE 1. The graph $D_{sd}(P_5, \{2\})$

i) consider two copies of G say G itself and G'
ii) if $u \in V(G)$ (first copy) then we denote the corresponding vertex as $u' \in V(G')$ (second copy)
iii) the vertex set of $D_{sd}(G, D_s)$ is $V(G) \cup V(G')$
iv) the edge set of $D_{sd}(G, D_s)$ is $E(G) \cup E(G') \cup E_{ds}$ where E_{ds} is the set of all edges (called the shadow distance edges) between two distinct vertices $u \in V(G)$ and $v' \in V(G')$ that satisfy the condition $d(u, v) \in D_s$ in G.

The image graph of a connected graph G [1], denoted by $I_{mg}(G)$, is the graph obtained by joining the vertices of the original graph G to the corresponding vertices of copy of G.

We recall the following results related to the edge domination number of a graph.

Theorem 1.1. [8] *An edge dominating set F is minimal if and only if for each edge $e \in G$, one of the following two conditions holds:*

i) $N(e) \cap F = \phi$
ii) there exists an edge $e \in E - F$ *such that* $N(e) \cap F = \{e\}$.

Theorem 1.2. [9] $\gamma'(C_n) = \lceil \frac{n}{3} \rceil$ *for* $n \geq 3$.

We begin our results with the edge domination number of the image graph of path graph and cycle graph.

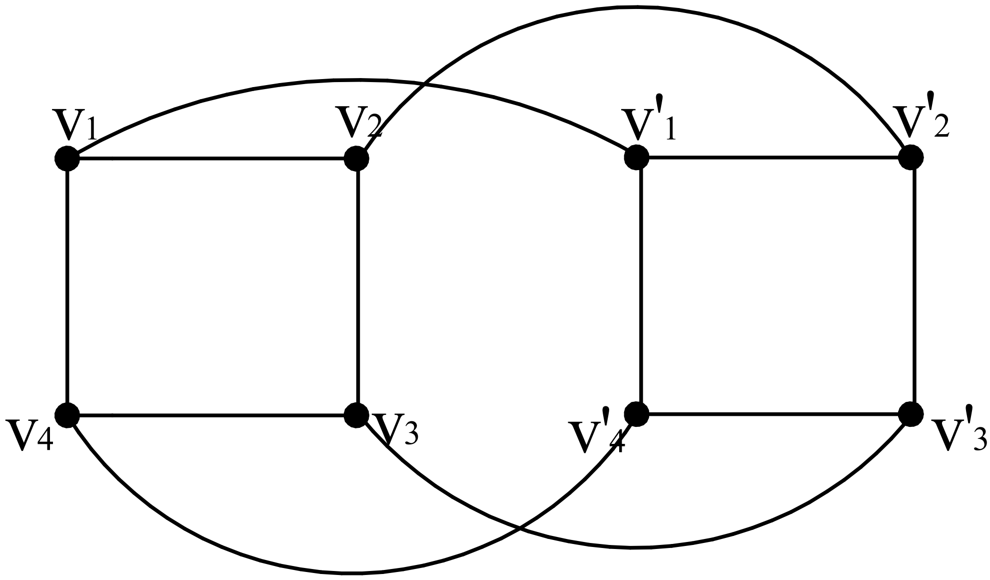

FIGURE 2. The image graph of cycle graph C_4

Theorem 1.3. *For $n \geq 2$, $\gamma^{'}(I_{mg}(P_n) = \lceil \frac{2n}{3} \rceil$.*

Proof. Let $G = I_{mg}(P_n)$. We consider the vertex set of G as $V(G) = \{v_i, v_j^{'}\}$, where $i, j = 1, 2, 3.....n$ and the edge set of G as $E(G) = \{E_1 \cup E_2 \cup E_3\}$, where $E_1 = \{e_1, e_2, e_3, ...e_{n-1}\}$ such that $e_i = (v_i, v_{i+1})$, where $i = 1, 2, ...n - 1$, $E_2 = \{e_1^{'}, e_2^{'}, e_3^{'}, ...e_{n-1}^{'}\}$ such that $e_i^{'} = (v_i^{'}, v_{i+1}^{'})$ where $i = 1, 2, ...n-1$, $E_3 = (v_i, v_i^{'})$, where $i = 1, 2, 3...n$.

For $n = 2$, the set $F = \{e_1, e_1^{'}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G) = 2$.

For $n = 3$, the set $F = \{e_1, e_2^{'}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G) = 2$.

For $n = 4$, the set $F = \{e_1, e_3, e_2^{'}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G) = 3$.

For $n = 5$, the set $F = \{e_1, e_3, e_2^{'}, e_4^{'}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G) = 4$. For $n = 6$, the set $F = \{e_1, e_3, e_2^{'}, e_5^{'}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G) = 4$.

For $n = 7$, the set $F = \{e_1, e_4, e_6, e_2^{'}, e_5^{'}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G) = 5$.

Let $n \geq 8$.

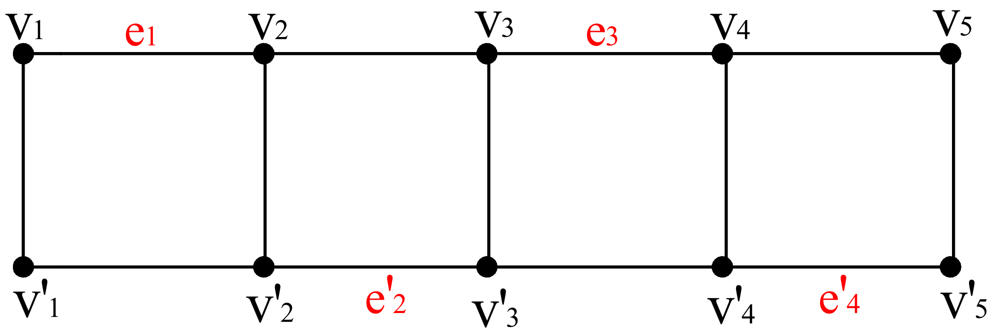

FIGURE 3. The image graph of path graph P_5

Consider the set $F = \begin{cases} F_1 \cup F_2 & n \equiv 0,2(mod3) \\ F_3 & n \equiv 1(mod3) \end{cases}$

where $F_1 = \{e_{3i-2}\}$, $1 \leq i \leq \lceil \frac{n}{3} \rceil$,

$F_2 = \begin{cases} \{e^{'}_{3j-1}\} \cup \{e^{'}_{n-1}\} & n \equiv 2(mod3) \\ e^{'}_{3k-1} & n \equiv 0(mod3) \end{cases}$, $1 \leq j \leq \lceil \frac{n}{3} \rceil - 1$, $1 \leq k \leq \frac{n}{3}$ and

$F_3 = \{\{e_{3i-2}\} \cup \{e_{n-1}\} \cup \{e^{'}_{3j-1}\}\}$, $1 \leq i, j \leq \lceil \frac{n}{3} \rceil - 1$

This set F is a minimal edge dominating set with minimum cardinality since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G. Hence, any set containing edges less than that of F cannot be a dominating set of G.

This implies that the set F described above is of minimum cardinality and since $|F| = \lceil \frac{2n}{3} \rceil$, it follows that $\gamma^{'}(G) = \lceil \frac{2n}{3} \rceil$

Hence the proof. □

Theorem 1.4. *For $n \geq 3$, $\gamma^{'}(I_{mg}(C_n)) = \lceil \frac{2n}{3} \rceil$.*

Proof. Let $G = I_{mg}(C_n)$. We consider the vertex set of G as $V(G) = \{v_i, v^{'}_j\}$, where $i, j = 1, 2, 3.....n$ and the edge set of G as $E(G) = \{E_1 \cup E_2 \cup E_3\}$, where $E_1 = \{e_1, e_2, e_3, ...e_n\}$such that $e_i = (v_i, v_{i+1})$ and $e_n = (v_n, v_1)$, where $1 \leq i \leq n-1$, $E_2 = \{e^{'}_1, e^{'}_2, e^{'}_3, ...e^{'}_n\}$ such that $e^{'}_i = (v^{'}_i, v^{'}_{i+1})$ and $e^{'}_n = (v_n, v_1)$ for $i = 1, 2, ...n-1$, $E_3 = (v_i, v^{'}_i)$, where $i = 1, 2, 3...n$.

Let $n \geq 3$.

Consider the set $F = \begin{cases} F_1 & n \equiv 0,2(mod3) \\ F_2 & n \equiv 1(mod3) \end{cases}$

where $F_1 = \{e_{3i-2}\} \cup \{e'_{3j-1}\}$, $1 \leq i, j \leq \lceil \frac{n}{3} \rceil$ and $F_2 = \{e_{3k-2}\} \cup \{e'_{3l-1}\} \cup \{v_{n-1}, v'_{n-1}\}$, $1 \leq k, l \leq \lfloor \frac{n}{3} \rfloor$

This set F is a minimal edge dominating set with minimum cardinality since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G. Hence, any set containing edges less than that of F cannot be a dominating set of G.

This implies that the set F described above is of minimum cardinality and since $|F| = \lceil \frac{2n}{3} \rceil$, it follows that $\gamma'(G) = \lceil \frac{2n}{3} \rceil$

Hence the proof. □

We now provide results related to the shadow distance graphs.

Theorem 1.5. *For $n \geq 2$, $\gamma(D_{sd}\{I_{mg}(P_n), \{2\}\}) = 2(\lceil \frac{2n}{3} \rceil)$.*

Proof. Consider two copies of $G = I_{mg}(P_n)$, one G itself and the other denoted by G'. Let $V(G) = \{v_i, v'_j\}$, where $i, j = 1, 2, 3, ...n$ and $E(G) = E_1 \cup E_2 \cup E_3$, where $E_1 = \{e_1, e_2, e_3, ...e_{n-1}\}$ such that $e_i = (v_i, v_{i+1})$, where $i = 1, 2, ...n-1$, $E_2 = \{e'_1, e'_2, e'_3, ...e'_{n-1}\}$ such that $e'_i = (v'_i, v'_{i+1})$ where $i = 1, 2, ...n-1$, $E_3 = (v_i, v'_j)$, where $i, j = 1, 2, 3...n$. Let $V(G') = \{v''_i, v'''_j\}$, where $i, j = 1, 2, 3, ...n$ and $E(G) = E_1 \cup E_2 \cup E_3$, where $E_1 = \{e''_1, e''_2, e''_3, ...e''_{n-1}\}$ such that $e_i = (v''_i, v''_{i+1})$, where $i = 1, 2, ...n-1$, $E_2 = \{e'''_1, e'''_2, e'''_3, ...e'''_{n-1}\}$ such that $e'_i = (v'''_i, v'''_{i+1})$ where $i = 1, 2, ...n-1$, $E_3 = (v''_i, v'''_i)$, where $i = 1, 2, 3...n$.

Let $G_1 = D_{sd}\{I_{mg}(P_n), \{2\}\}$

For $n = 2$, the set $F = \{e_1, e'_1, e''_1, e'''_1\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma'(G_1) = 4$.

Let $n \geq 3$.

$$\text{Consider the set } F = \begin{cases} F_1 & n \equiv 0 (mod 3) \\ F_2 & n \equiv 1 (mod 3) \\ F_3 & n \equiv 2 (mod 3) \end{cases}$$

where $F_1 = \{e_{3j-2}\} \cup \{e''_{3j-2}\} \cup \{e'_{3j-1}\} \cup \{e'''_{3j-1}\}$, $1 \leq j \leq \frac{n}{3}$,

$F_2 = \{e_{3i-2}\} \cup \{e_{n-1}\} \cup \{e''_{3i-2}\} \cup \{e''_{n-1}\} \cup \{e'_{3j-1}\} \cup \{e'''_{3j-1}\}$, $1 \leq i \leq \frac{n-1}{3}$, $1 \leq j \leq \lfloor \frac{n}{3} \rfloor$

$F_3 = \{e_{3i-2}\} \cup \{e''_{3i-2}\} \cup \{e'_{3j-1}\} \cup \{e'''_{3j-1}\} \cup \{e'_{n-1}\} \cup \{e'''_{n-1}\}$, $1 \leq i \leq \frac{n+1}{3}$, $1 \leq j \leq \frac{n-2}{3}$

This set F is a minimal edge dominating set with minimum cardinality since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G_1. Hence, any set containing edges less than that of F cannot be a dominating set of G_1.

This implies that the set F described above is of minimum cardinality and since $|F|$ =2($\lceil \frac{2n}{3} \rceil$), it follows that $\gamma'(G_1)$= 2($\lceil \frac{2n}{3} \rceil$)

Hence the proof. □

Theorem 1.6. *For $n \geq 3$, $\gamma(D_{sd}\{I_{mg}(P_n), \{3\}\}) = 2(\lceil \frac{2n}{3} \rceil)$.*

Proof. Consider two copies of G= $I_{mg}(P_n)$. The vertex set and edge set of G are as in theorem 1.5.

Let $n \geq 3$.

$$\text{Consider the set } F = \begin{cases} F_1 & n \equiv 0(mod3) \\ F_2 & n \equiv 1(mod3) \\ F_3 & n \equiv 2(mod3) \end{cases}$$

where $F_1 = \{e_{3j-2}\} \cup \{e'''_{3j-2}\} \cup \{e'_{3k-1}\} \cup \{e''_{3k-1}\}$, $1 \leq j \leq \lceil \frac{n}{3} \rceil$, $1 \leq k \leq \lfloor \frac{n}{3} \rfloor$

$F_2 = \{e_{3j-2}\} \cup \{e_{n-1}\} \cup \{e'''_{n-1}\} \cup \{e'''_{3j-2}\} \cup \{e'_{3k-1}\} \cup \{e''_{3k-1}\}$, $1 \leq j \leq \frac{n-1}{3}$, $1 \leq k \leq \lfloor \frac{n}{3} \rfloor$

$F_3 = \{e_{3j-2}\} \cup \{e'''_{3j-2}\} \cup \{e'_{3k-1}\} \cup \{e'_{n-1}\} \cup \{e''_{3k-1}\} \cup \{e''_{n-1}\}$, $1 \leq j \leq \lceil \frac{n}{3} \rceil$, $1 \leq k \leq \frac{n-2}{3}$

This set F is a minimal edge dominating set with minimum cardinality since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G_1. Hence, any set containing edges less than that of F cannot be a dominating set of G_1.

This implies that the set F described above is of minimum cardinality and since $|F| = 2(\lceil \frac{2n}{3} \rceil)$, it follows that $\gamma'(G_1)$= 2($\lceil \frac{2n}{3} \rceil$)

Hence the proof.

□

Theorem 1.7. *For $n \geq 3$,*

$$\gamma(D_{sd}\{I_{mg}(C_n), \{2\}\}) = \begin{cases} 2n, & n = 3, 4 \\ 2n - p, & n \geq 5, \quad 6j-1 \leq n \leq 6j+4, p = 2j, j = 1, 2, 3, ...n \end{cases}.$$

Proof. Consider two copies of $G = I_{mg}(C_n)$, one G itself and the other denoted by G'. Let $V(G) = \{v_i, v_j'\}$, where $i, j = 1, 2, 3, ...n$ and $E(G) = E_1 \cup E_2 \cup E_3$, where $E_1 = \{e_1, e_2, e_3, ...e_n\}$ such that $e_i = (v_i, v_{i+1})$ and $e_n = (v_n, v_1)$, where $1 \leq i \leq n-1$, $E_2 = \{e_1', e_2', e_3', ...e_n'\}$ such that $e_i' = (v_i', v_{i+1}')$ and $e_n' = (v_n, v_1)$ for $i = 1, 2, ...n-1$, $E_3 = (v_i, v_j')$, where $i, j = 1, 2, 3...n$. Let $V(G') = v_i'', v_j'''$, where $i, j = 1, 2, 3, ...n$ and $E(G) = E_1 \cup E_2 \cup E_3$, where $E_1 = \{e_1'', e_2'', e_3'', ...e_n''\}$ such that $e_i = (v_i'', v_{i+1}'')$ and $e''(v_n'', V_1'')$, where $i = 1, 2, ...n-1$, $E_2 = \{e_1''', e_2''', e_3''', ...e_n'''\}$ such that $e_i' = (v_i''', v_{i+1}''')$ and $e'''(v_n''', V_1''')$, where $i = 1, 2, ...n-1$, $E_3 = (v_i'', v_i''')$, where $i = 1, 2, 3...n$.

Let $G_1 = (D_{sd}\{G, \{2\}\}$.

case(i): Let $n = 6j-3$, $6j+1$, where $j = 1, 2, 3, ...$

Consider the set $F = \{e_{2i}\} \cup \{e_{2k-1}'\} \cup \{e_{2k-1}'''\} \cup \{e_{2i}''\}$ where $1 \leq i \leq \lfloor \frac{n}{2} \rfloor$, $1 \leq k \leq \lceil \frac{n}{2} \rceil$

case(ii): Let $n = 6j-1$, where $j = 1, 2, 3, ...$

Consider the set $F = \{e_{2i-1}\} \cup \{e_{2k}'\} \cup \{e_{2i-1}'''\} \cup \{e_{3l-1}''\} \cup \{e_n''\}$ where $1 \leq i, k \leq \lfloor \frac{n}{2} \rfloor$, $1 \leq l \leq \lfloor \frac{n-1}{3} \rfloor$

case(iii): Let $n = 6j-2$, where $j = 1, 2, 3, ...$

Consider the set $F = \{e_{2i-1}\} \cup \{e_{2k-1}'\} \cup \{e_{3l-1}''\} \cup \{e_{3m-1}'''\} \cup \{e_n''\}$ where $1 \leq i, k \leq \frac{n}{2}$, $1 \leq l \leq \lceil \frac{n-1}{3} \rceil$, $1 \leq m \leq \lceil \frac{n+1}{3} \rceil$

case(iv): Let $n = 6j$, where $j = 1, 2, 3, ...$

Consider the set $F = \{e_{2i-1}\} \cup \{e_{2k-1}'\} \cup \{e_{3l-1}''\} \cup \{e_{3m-1}'''\}$ where $1 \leq i, k \leq \frac{n}{2}$, $1 \leq l \leq \lceil \frac{n-1}{3} \rceil$, $1 \leq m \leq \lfloor \frac{n-1}{2} \rfloor$

case(v): Let $n = 6j+2$, where $j = 1, 2, 3, ...$

Consider the set $F = \{e_{2i-1}\} \cup \{e_{2k-1}'\} \cup \{e_{3l-1}''\} \cup \{e_{3m-1}'''\}$ where $1 \leq i, k \leq \frac{n}{2}$, $1 \leq l \leq \lceil \frac{n-1}{3} \rceil$, $1 \leq m \leq \frac{n+1}{3}$

The set F in cases $(i), (ii), (iii), (iv)$ and (v) is a minimal edge dominating set with minimum cardinality since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G_1. Hence, any set containing edges less than that of F cannot be a dominating set of G_1. Also G_1 is regular $(n \geq 5)$ of degree 14 and each edge of G_1 is of degree 14 and an edge of G_1 can dominate atmost 15 distinct edges of G_1 including itself.

This implies that the set F described above is of minimum cardinality and since $|F|$

$$= \begin{cases} 2n, & n = 3, 4 \\ 2n - p, & n \geq 5, \quad 6j - 1 \leq n \leq 6j + 4, p = 2j, j = 1, 2, 3, ...n \end{cases},$$

it follows that $\gamma^{'}(D_{sd}\{G, \{2\}\}) = \begin{cases} 2n, & n = 3, 4 \\ 2n - p, & n \geq 5, \quad 6j - 1 \leq n \leq 6j + 4, p = 2j, j = 1, 2, 3, ...n \end{cases}$.

Hence the proof. □

Theorem 1.8. *For $n \geq 4$,*

$$\gamma(D_{sd}\{I_{mg}(C_n), \{3\}\}) = \begin{cases} \frac{4n+6}{3}, & n \equiv 0(mod3) \\ 4\lceil \frac{n}{3} \rceil, & n \equiv 1, 2(mod3) \end{cases}.$$

Proof. Consider two copies of $G = I_{mg}(C_n)$. The vertex set and edge set of G are as in theorem 1.7.

Let $G_1 = (D_{sd}\{G, \{3\}\}$.

Let $n \geq 4$.

Consider the set $F = \begin{cases} F_1 & n \equiv 0(mod3) \\ F_2 & n \equiv 1(mod3) \\ F_3 & n \equiv 2(mod3) \end{cases}$

where $F_1 = \{e_{3i-2}\} \cup \{e_n\} \cup \{e^{'''}_{3i-2}\} \cup \{e^{'''}_n\} \cup \{e^{'}_{3j-1}\} \cup \{e^{''}_{3j-1}\}$, $1 \leq i, j \leq \frac{n}{3}$,

$F_2 = \{e_{3i-2}\} \cup \{e^{'''}_{3i-2}\} \cup \{e^{'}_{3j-1}\} \cup \{e^{''}_{3j-1}\}$, $1 \leq i, j \leq \frac{n+2}{3}$,

$F_3 = \{e_{3i-2}\} \cup \{e^{'''}_{3i-2}\} \cup \{e^{'}_{3j-1}\} \cup \{e^{''}_{3j-1}\}$, $1 \leq i, j \leq \frac{n+1}{3}$

This set F is a minimal edge dominating set with minimum cardinality since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G_1. Hence, any set containing edges less than that of F cannot be a dominating set of G_1. Also G_1 is regular $(n \geq 7)$ of degree 12 and each edge of G_1 is of degree 12 and an edge of G_1 can dominate atmost 13 distinct edges of G_1 including itself.

This implies that the set F described above is of minimum cardinality and since $|F|$

$$= \begin{cases} \frac{4n+6}{3}, & n \equiv 0(mod3) \\ 4\lceil \frac{n}{3} \rceil, & n \equiv 1, 2(mod3) \end{cases},$$

it follows that $\gamma^{'}(D_{sd}\{G,\{3\}\}) = \begin{cases} \frac{4n+6}{3}, & n \equiv 0(mod3) \\ 4\lceil\frac{n}{3}\rceil, & n \equiv 1,2(mod3) \end{cases}$.

Hence the proof.

□

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